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Limiting rival's efficiency via conditional discounts*

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Abstract

This paper studies the impact of a dominant firm's conditional discounts on competitors' learning-by-doing. In a vertical context where a dominant upstream supplier and a competitive fringe sell their products to a single downstream firm, we analyze whether the dominant supplier prefers to offer a discount scheme, as in particular a quantity or market-share discount. In a dynamic setting with complete information and learning-by-doing, short-term market-share discounts and long-run contracts are more profitable to the dominant supplier than simple two-part tariffs or quantity discounts. We show that two-part tariffs as well as quantity discounts lead to more learning than market-share discounts, or long-term contracts. Thus, the dominant firm's contract choice restricts the competitive fringe's efficiency gain. Similar results occur for network effects.

Keywords: Market-share discounts, quantity discounts, learning-by-doing, dominant upstream supplier, competitive fringe.

JEL classification numbers: L13, L42

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1 Introduction

Offering conditional discounts is a standard practice in supply chains. Upstream suppliers grant discounts to customers if their purchasing volume achieves or exceeds certain thresholds. In particular these thresholds can be based on quantity targets, defining so-called quantity discounts, or percentages of total requirements, i.e. market-share discounts (Geradin, 2009; Faella, 2008; Ahlborn & Bailey, 2006). The significant difference between these discounts is actually given by the types of thresholds: In case of quantity discounts, the seller granting the discount offers lower prices only if the buyer purchases at least the given quantity threshold. That is, these contracts have a direct influence on the sales level of the discount-granting firm. In contrast, market-share discounts are offered, when at least $\rho \cdot 100\%$ of the buyer's aggregate purchase is made by the discount-granting firm. Hence, market-share discounts have an (indirect) influence on all manufacturers' sales. They influence relative sales levels of competitors, but they cannot influence direct sales levels.

The impacts of these discounts on competitors, downstream firms, and final consumers vary: Besides pro-competitive reasons such as stimulating demand and consumer surplus, conditional discounts can also have anti-competitive effects.[1] When granted by a dominant supplier, antitrust authorities claim that discount schemes characterize abusive pricing practices as they can be loyalty-inducing, lead to market foreclosure or consumer harm.[2] In the Intel Decision[3], for example, the European Commission identified Intel's conditional discounts to present an illegal practice in the x86 CPU market. In particular, the pricing practices of Intel which is the dominant manufacturer in this market, allegedly restricted AMD in competition/innovation incentives, and reduced consumer choice, from 2002 to 2007.

Furthermore, the European Commission emphasized the exclusionary effects of conditional discounts and rebates in its Guidance Paper (2009). In relation to foreclosure effects in general, it states that exclusionary practices are not only those that exclude more or equally efficient competitors but also those that restrict less efficient competitors, because *'in the absence of an abusive practice such a competitor may benefit from demand-related advantages, such as network and learning effects, which will tend to enhance its efficiency'*.[4]

In that regard, the question arises whether a specific discount scheme is more suitable to maximize dominant supplier's profits and/or to restrict the competitors' learning effects. As market-share discounts affect the relative purchase levels of competitors, they might represent better means to restrict the rivals' efficiency gain. However, market-share discounts do not influence total sales levels. Therefore, it is not obvious whether these discount schemes are more profitable than quantity discounts, which affect direct sales levels for the dominant

[1] For an overview of the effects of loyalty rebates, see Inderst and Schwalbe (2008), for quantity as well as market-share discounts see the literature review.

[2] See for example Waelbroeck (2005), Hovenkamp (2006), Tom, Balto, and Averitt (1999) and Greenlee and Reitman (2005).

[3] *Intel Commission Decision, 2009, Case COMP/C-3/37.990.*

[4] EC Guidance Paper (2009), paragraph 24.

supplier's good.

In this paper, we examine the contract choice of a dominant upstream firm when its rival faces learning-by-doing, as well as network effects. In our two-period model, the dominant upstream firm and its competitors sell their goods to a single downstream firm. We concentrate on the dominant firm's contracting decision, where we especially allow for these contractual terms which are often claimed to be anti-competitive. That is, we investigate (short term) market-share discounts, quantity discounts, and also consider long-term contracts where contractual terms for both periods are set in the first period.[5] To highlight the potential differences in outcomes, we also analyze simple two-part tariffs, offered by the dominant firm.

In our setting, we suppose that there is one specific good which is solely produced by the dominant supplier. All other firms in the upstream market produce an imperfect substitute of this good. Assuming fierce Bertrand competition, and learning-by-doing for this alternative good leads to the fact that in each period only the firm with lowest cost is active. Further simplifying this assumption, we consider a competitive fringe that faces learning-by-doing effects.[6] In the downstream market, there is only one active firm. As we suppose complete information, this monopolistic firm maximizes profits, anticipating learning-by-doing of upstream suppliers.

As a first, general result, we show that a dominant supplier's contract decision is influenced by the competitive fringe's learning-by-doing effects. While the dominant supplier is indifferent between short-term two-part tariffs and discounts, when learning-by-doing is not taking place, it prefers only market-share discounts when learning-by-doing occurs. To be precise, the dominant supplier achieves maximum profits, given by the joint-profit maximizing outcome, when it grants short-term market-share discounts in both periods.[7] By offering two-part tariffs or quantity discounts in period 1, the dominant supplier would earn lower profits. In this regard, our model provides a novel explanation for the use of market-share discounts as opposed to no discounts or quantity discounts.

Furthermore, we show that long-run contracts, which initially specify the pricing scheme for all periods, are not affected by the competitive fringe's learning effects: All long-term contracts lead to the same maximum profits for the dominant supplier. In particular, long-term contracts are preferable for the dominant supplier as they lead to the joint-profit maximizing outcome and therefore to maximum profits for the dominant upstream firm.

In this context, we find that even though the competitive fringe's learning effects represent a competitive threat to the dominant supplier, this supplier will not use conditional discounts to exclude its rival.[8] Instead, the joint-profit maximizing outcome (and learning

[5]For an overview see Faella (2008), p.386; Hovenkamp (2006), footnote 25; Discussion Paper (2005), paragraph 158. In addition, the use of conditional discounts over longer time periods are addressed in Concord Boat v. Brunswick, 2000, 207 F.3d 1039., for example.

[6]This assumption simplifies calculations and allows for the above mentioned analysis.

[7]Granting market-share discounts in period 1, and no discounts or quantity discounts in period 2 will also lead to this optimal result for this upstream firm.

[8]As we consider the rival to be a competitive fringe, exclusion means that the sales of the fringe are zero.

effects) occurs, because the dominant supplier's preferred contracts shift additional downstream rents upstream. Nevertheless, short-term quantity discounts and two-part tariffs lead to lower profits for the dominant supplier, but they induce larger learning effects for the competitive fringe. Therefore, market-share discounts restrict the rivals' learning effects, and hence rivals' efficiency gains.

Related Literature

First of all, our model is related to the economic literature on market-share and quantity discounts that explains pro- and anticompetitive effects of these terms.

A first model that examines the exclusionary effects of loyalty discounts is given by Ordover and Shaffer (2007). In their two-period model, there are two active sellers and a single buyer who prefers to purchase from both upstream firms. When at least one seller is financially constrained, equilibria exist in which loyalty discounts induce exclusion of this seller. However, in their model, Ordover and Shaffer argue that when excluding its rival, the discount-granting seller prices below cost and earns less long-run profits than without exclusion. In a framework with an incumbent firm and a potential entrant, Erutku (2006) shows that rebates can lead to exclusion. Furthermore, Packalen (2011) and Chen and Shaffer (2010) both build on the naked-exclusion framework by Rasmusen, Ramseyer, and Wiley (1991). In their models, they show that the incumbent can deter entry of a potential entrant by offering market-share discounts. Our model relates to the literature as we focus on profit maximizing contracts. Exclusion or hinderance of rivals is not the first aim. We examine whether profit-maximizing contracts have impacts on rivals, regardless of exclusionary intent.

As mentioned earlier, there are pro-competitive reasons for conditional discounts. Mills (2010) and Sloev (2010) show, for example, that market-share discounts can induce selling effort as well as innovation incentives in the downstream market. In addition, double marginalization can be eliminated by the use of conditional discounts. Inderst and Shaffer (2010) argue that market-share discounts lead to the joint profit maximizing outcome in a 2×2-framework. Kolay, Shaffer, and Ordover (2004) create a simple vertical structure where a single upstream firm sells its good to a monopolistic downstream firm. By offering all-units discounts without charging a fixed fee, double marginalization is eliminated. Our model relates to these papers as profit-maximizing contract terms eliminate double marginalization, even in the dynamic context with learning and network effects.

Furthermore, Kolay et al. (2004) also show that all-units discounts lead to maximum profits for the upstream monopolist when there is asymmetric information about demand. Their result stems from the additional use of discount menus. Without contract menus but with simple discount schemes, recent studies claim that primarily market-share discounts represent a profitable alternative for dominant upstream firms when there is uncertainty about demand.[9] In our framework, we do not allow for demand or cost uncertainties. Nev-

[9] Cf. Chioveanu and Akgun (2011) and Majumdar and Shaffer (2009). In addition, Calzolari and Denicolo

ertheless, our paper contributes to this literature as it shows that even without uncertainty, there are potential cases where market-share discounts are more profitable than different discount terms. Thus, our model presents an additional reason why dominant firms may specifically offer market-share discounts.

Karlinger (2008) further extends the literature on rebate schemes in addressing network effects. An incumbent produces and offers a network good to a number of asymmetric buyers when a more efficient rival tries to enter the market. The article shows inter alia that the incumbent is more likely to exclude the rival due to the advantages by network externalities which the rival does not face. In our model, however, the rival is already an active firm and faces larger learning or network effects than the dominant upstream firm. Thus, inter-temporal effects present a threat to the dominant firm, rather than an advantage.

To the best of our knowledge, there is no article dealing with learning-by-doing in the context of conditional discounts. Furthermore, there is no article addressing the question whether profitable discount schemes may partially restrict a dominant firm's rival in its efficiency gain. Our paper contributes to the literature as it analyzes the impact of a dominant firm's discount schemes on a rival's learning-by-doing and network effects.

Outline

The article is structured as follows. Section 2 describes the framework. In Section 3, we solve the model for the case that inter-temporal externalities are not present. Here, short-run, and long-run contracts are examined where we consider simple two-part tariffs, quantity and market-share discounts. The profit-maximizing contract terms in case of learning-by-doing are solved for in Section 4. Note that we do not evaluate welfare implications. In Section 5, some further extensions of the model are studied. Section 6 concludes.

2 Framework

We analyze conditional discounts in a supplier-retailer framework. A monopolistic downstream firm R purchases products of the upstream suppliers and resells them to final consumers. There are two types of products, namely a specific good produced by the dominant upstream firm M only, and an imperfect substitute manufactured by a competitive fringe C. The contractual terms between the downstream firm and the dominant supplier M is the focus of our analysis.

We suppose there are two periods. Upstream firms M and C produce goods with constant marginal cost. While M has cost $c > 0$ in each period, the competitive fringe might learn over time. It has marginal cost c_1 in period 1 and $c_2 < c_1$ in period 2, defined by

$$c_2 = \max\{0, c_1 - \lambda q_{C1}\}.$$

(2009) and Calzolari and Denicolò (2011) analyze principal-agency models where a seller can use quantity discounts, or market-share discounts for anticompetitive reasons.

where q_{C1} is the quantity sold by the competitive fringe C in period 1, and $\lambda \geq 0$ characterizes the learning parameter.[10]

Marginal costs in period 2 depend on the fringe's learning-by-doing effect. Analogue to Cabral and Riordan (1997), we suppose the learning curve to be proportional to quantities sold in the first period.
The monopolistic downstream firm R purchases quantities q_{Mt}, q_{Ct} in period t. The downstream firm resells the products to final consumers where the inverse demand system in period $t = 1, 2$ is given by

$$P_M(q_{Mt}, q_{Ct}),$$
$$P_C(q_{Mt}, q_{Ct}).$$

We assume that $P_J(q_{Mt}, q_{Ct}) \in \mathcal{C}^1$ and $\frac{\partial P_J}{\partial q_{Jt}} < \frac{\partial P_J}{\partial q_{It}} < 0$ whenever $P_J(q_{Mt}, q_{Ct}) > 0$ for $J, I \in \{M, C\}$.[11] Thus, the impact of J's good on its final price is larger than the impact of the imperfect substitute on this price. Industry profits are assumed to be quasi concave and maximized by non-negative values.

The competitive fringe C is not capable of acting strategically. As the competition on its good is fierce, the wholesale price equals marginal cost $w_{Ct} = c_t$. The dominant supplier M however charges wholesale prices w_{Mt} and a fixed fee F_t for $t = 1, 2$. We concentrate on the contracts between M and downstream firm R.
First of all, we distinguish between short-term and long-term contracts. Short-term contracts offered by the dominant supplier apply to a single period. In this case, M offers a contract in period 1 which specifies wholesale prices and a fixed fee. Afterwards, it offers a second contract in period 2. In contrast, in case of a long-term contract, all contract terms (for the first *and* second period) are defined in the first period. Hence there is only one offer which defines prices for both periods.

For both short- and long-term contracts, we suppose that the dominant upstream firm can choose a two-part tariff (2PT), quantity discounts (Q), or market-share discounts (MS) where each contract with (and without) discount schemes additionally defines fixed fees F_t. M's wholesale prices in case of quantity and market-share discounts w^{Q}_{Mt}, w^{MS}_{Mt} are characterized by

$$w^{\mathrm{Q}}_{Mt} = \begin{cases} w_{\mathrm{Q}t}, & \text{if } q_{Mt} \geq q^*_{Mt} \\ w_t, & \text{if } q_{Mt} < q^*_{Mt} \end{cases}, \text{ and } \quad w^{\mathrm{MS}}_{Mt} = \begin{cases} w_{\mathrm{MS}t}, & \text{if } \frac{q_{Mt}}{q_{Mt}+q_{Ct}} \geq \rho^*_t \\ w_t, & \text{if } \frac{q_{Mt}}{q_{Mt}+q_{Ct}} < \rho^*_t \end{cases}.$$

The respective threshold levels are q^*_{Mt} which corresponds to the quantity threshold for the quantity discount and ρ^*_t, that is the percentage R has to purchase for paying the lower price $w_{\mathrm{MS}t}$ in case of a market-share discount.

[10] Note that these assumptions imply that the competitive fringe might be more, less or equally efficient as the dominant upstream firm.

[11] We suppose further that in case of two-times differentiable functions, the cross-derivatives are negative, that is $\frac{\partial^2 P_J}{\partial q_{Jt} \partial q_{It}} \leq 0$, with $\frac{\partial^2 P_J}{\partial q_{Jt} \partial q_{It}} = \frac{\partial^2 P_J}{\partial q_{It} \partial q_{Jt}}$ for $J \neq I$ and, additionally, $\frac{\partial^2 P_J}{\partial q^2_{Jt}} \leq 0$ as well as $|\frac{\partial^2 P_I}{\partial q_{Mt} \partial q_{Ct}}| \leq |\frac{\partial^2 P_I}{\partial q^2_{Ct}}|$.

Timing of the game

The timing of the two-period game is as follows. In period one, the dominant upstream firm offers a contract which is either a short-term contract for period 1 or a long-term contract (for both periods). The competitive fringe sells its goods at a price of c_1. The downstream firm then accepts or rejects the offer, purchases and resells the products[12] for the first period. Once the first-period purchase is completed, learning-by-doing occurs.

When the dominant firm's short-term contract offer in period 1 is accepted by the downstream firm, M will offer a second contract in period 2. The downstream firm decides whether to accept or reject this second offer and will set final prices for period 2. If the dominant supplier offered a long-term contract, second-period contract terms are already fixed and the downstream firm directly sets prices.

The structure of demand and costs is common knowledge to downstream and upstream firms. Both the dominant upstream supplier and the downstream firm anticipate C's learning-by-doing and maximize profits. The competitive fringe in contrast is restricted to zero profits by definition.

3 Benchmark case: no learning

We first examine the case where learning-by-doing effects do not occur. This is either the case because these effects do not appear in a specific market, or because they are not observable, let alone verifiable. In the following, short-term and long-term contracts are analyzed when the learning parameter λ equals zero.

3.1 Short-term contracts

If the dominant upstream firm offers short-term contracts, a single-period analysis is sufficient to derive the profit-maximizing contract: Solving the benchmark model by backwards induction, the optimal decision in period 2 is repeated in period 1 because the optimization problems (for both periods) are the same. Therefore, the comparison of a single-period two-part tariff with a quantity discount and a market-share discount already shows the profit maximizing contract choice of the dominant supplier, when learning does not occur.

When the dominant supplier offers a simple two-part tariff $(w_t^{2\text{PT}}, F_t^{2\text{PT}})$ in a single-period model (t=1,2), the downstream firm decides whether to accept or reject the contract offer according to its related profits. If the two-part tariff is accepted, the downstream firm decides upon quantities q_{Mt}, q_{Ct} according to the wholesale price w_t. The optimal downstream gross profit is then given by

$$\pi_{Rt}(q_{Mt}, q_{Ct}) = (P_M(q_{Mt}, q_{Ct}) - w_{Mt})q_{Mt} + (P_C(q_{Mt}, q_{Ct}) - c_1)q_{Ct} \tag{1}$$

[12]Depending on the accepted contract, the downstream firm sells either both goods if it accepted or only the fringe's good if it rejected the dominant supplier's offer.

where $q_{Mt} = q_{Mt}(w_t)$ and $q_{Ct} = q_{Ct}(w_t)$ are the profit maximizing quantity levels.[13]

If instead the downstream firm rejects the contract, it earns the outside option $\pi^o_{Rt}(c_1)$, given by

$$\pi^o_{Rt}(c_1) = \max_{q_{Ct}}(P_C(0, q_{Ct}) - c_1)q_{Ct}. \tag{2}$$

Maximizing own profits $\pi_{Mt}(w_t) = q_{Mt}(w_t)(w_t - c) + F_t$, the dominant upstream firm anticipates R's quantity choice and participation constraint, which is

$$\pi_{Rt}(q_{Mt}(w_t), q_{Ct}(w_t)) - F_t \geq \pi^o_{Rt}(c_1),$$

as the downstream firm only accepts the two-part tariff when its related profits exceed, or at least equal its outside option.

In the profit-maximizing case, the participation constraint is binding, that is $F_t = \pi_{Rt}(q_{Mt}(w_t), q_{Ct}(w_t)) - \pi^o_{Rt}(c_1)$. Thus, (additional) rents from purchasing both goods are shifted upwards by the use of F_t. The wholesale price is then used to maximize these downstream profits. When the dominant supplier sets the wholesale price larger than marginal cost, it earns a positive mark-up. Yet, as the fixed fee shifts downstream profits upstream and these profits are larger when the price is smaller, a price larger than marginal cost is not optimal. When in contrast, the wholesale price is lower than marginal cost, the downstream firm's profits are large and promise additional benefits for the dominant supplier. However, a lower wholesale price, thus below-cost pricing, also means a loss as the dominant supplier's profits decrease when price minus costs decrease. When the wholesale price equals marginal cost, $w_t = c$, the downstream firm maximizes industry profits

$$\pi^I_t(c_1) = max_{q_{Mt}, q_{Ct}} \; (P_M(q_{Mt}, q_{Ct}) - c)q_{Mt} + (P_C(q_{Mt}, q_{Ct}) - c_1)q_{Ct}$$

where index I represents the joint-profit maximizing results. The joint-profit maximizing quantity levels are given by q^I_{Mt} and q^I_{Ct}. Hence, the profit-maximizing two-part tariff is defined by $w^{\text{2PT}}_t = c$ and $F^{\text{2PT}}_t = \pi_{Rt}(q_{Mt}(c), q_{Ct}(c)) - \pi^o_{Rt}(c_1)$. In this way, the dominant upstream firm earns $\pi^{\text{2PT}}_{Mt}(c_1) = \pi^I_t(c_1) - \pi^o_{Rt}(c_1)$ and the downstream firm $\pi^{\text{2PT}}_{Rt}(c_1) = \pi^o_{Rt}(c_1)$, in period $t = 1, 2$.

Accordingly, simple two-part tariffs already lead to maximum profits for the dominant upstream firm. Larger upstream profits than given by the joint-profit maximizing outcome (that is industry profits minus the outside option $\pi^o_{Rt}(c_1)$) are not feasible because the dominant supplier needs to leave $\pi^o_{Rt}(c_1)$ to the downstream firm R, to achieve R's acceptance.

Therefore, granting conditional discounts does not improve the already best result for the dominant upstream firm. Defining additional discount conditions leads to the same result as the profit-maximizing two-part tariff.[14] Hence, the dominant supplier is indifferent between

[13] As there is no change in the competitive fringe's costs, the fringe faces marginal cost c_1 in both periods. Note that the downstream firm's net profits are given by $\pi_{Rt}(q_{Mt}, q_{Ct}) - F_t$.

[14] In particular, the profit-maximizing quantity discounts are given by $q^*_{Mt} = q^I_{Mt}$, an unattractively large un-discounted price w_t, and the discounted price w_{Qt} as well as fixed fee F_t given by $max_{q_{Ct}} \; \pi_{Rt}(q^*_{Mt}, q_{Ct}) = \pi^o_{Rt}(c_1)$. The (single) profit-maximizing market-share discount is given by $\rho^*_1 = \frac{q^I_{Mt}}{q^I_{Mt}+q^I_{Ct}}$, an unattractively large w_t, the discounted wholesale price $w_{\text{MS}t} = c$, and the fixed fee $F^{\text{MS}}_t = \pi^I_2(c_2) - \pi^o_{Rt}(c_2)$.

short-term two-part tariffs, quantity discounts and market-share discounts.

3.2 Long-term contracts

If the dominant upstream firm offers a long-run contract, this contract defines contract terms for both periods. It includes a simple wholesale price, optionally discount conditions for both periods as well as a fixed fee in either both or only one period. As the dominant supplier can specify the contract terms defined in the short-term contracts in form of a long-term contract, these contracts also lead to the joint-profit maximizing outcome, and maximum profit for the dominant supplier.

Proposition 1 (Without learning-by-doing).
When learning-by-doing does not occur, the dominant upstream supplier is indifferent between short-term or long-term contracts, and indifferent between two-part tariffs, quantity discounts or market-share discounts. These contracts lead to the joint-profit maximizing outcome, and therefore to maximum profits for the dominant supplier.

Most notably this result is driven by two assumptions. First of all, the joint profit-maximizing outcome results due to the considered setting which is similar to a common-agency model. We assume there are (at least) two upstream firms dealing with a single, common agent. In the standard common-agency model, both firms, maximizing profits, set wholesale prices equal to marginal costs and charge fees, which accounts for maximum industry profits. Hence, the upstream firms share maximum joint profits. In our model, the wholesale price of one of these upstream firms, namely the competitive fringe's price, is already equal to marginal cost. By setting $w_{Mt} = c$, the dominant upstream firm reaches the joint-profit maximizing outcome where the downstream firm R and the dominant upstream firm share joint profits in accordance with R's outside option.

Furthermore, Proposition 1 depends on the assumption on fixed fees: For all contracts, we assumed that the dominant supplier charges a fixed fee. This fee however represents a second instrument for the dominant supplier, besides the wholesale price and optional discount conditions. As a simple two-part tariff already achieves the profit maximizing, best result for the dominant supplier, any further specifications of this contract serve as an over-specification and cannot improve the already best outcome, in the dominant supplier's view. All contracts lead to the same, joint profit-maximizing results. Therefore, in this setting, it is ambiguous why the dominant supplier should offer discount schemes when the two-part tariff is already optimal.

When, in contrast, fixed fees are infeasible, a linear contract would not lead to maximum industry profits. Then, conditional discounts can improve upstream profits as they characterize an additional instrument to control for downstream profits.

In the following, we introduce learning-by-doing by the competitive fringe to analyze the influence of this competitive threat on the contract decision of the dominant supplier.

4 Learning-by-doing

Learning-by-doing necessitates a multi-period model. For simplicity, we consider a model with two periods where the rivals of the dominant supplier face learning effects. As we summarize M's rivals by considering the competitive fringe C, this means that C's marginal cost decrease over time. While M's cost are constantly given by $c > 0$, we suppose that the fringe's cost in period 1 are given by $c_1 > 0$. For the second period however, learning reduces its marginal cost proportionally to the learning parameter λ and the quantity q_{C1}, sold in period 1.[15] We suppose here, that $c_2(q_{C1}) = c_1 - \lambda q_{C1} > 0$, respectively $\lambda < \frac{c_1}{q^o_{C1}}$, where $q^o_{C1} > 0$ is the quantity sold of C's good, when the downstream firm only deals with the competitive fringe.[16]

Learning-by-doing is assumed just for the competitive fringe.[17] Yet, as the fringe always earns zero profits, by definition, C's learning curve mainly concerns the downstream firm: As downstream firm R knows about the fringe's learning-by-doing effect, it will purchase more of C's good in period 1 anticipating the increase in profits initiated by the lower wholesale price in period 2. The dominant upstream supplier anticipates R's choice and maximizes profits by choosing contractual terms.

Solving for M's optimal contracts, we compare both the long and short-term contracts which specify a fixed fee and wholesale prices, and potentially include conditional discounts.

4.1 Short-term contracts

We consider first the case where the dominant upstream firm decides about profitable short-run contracts. That is, we suppose that the dominant supplier offers a single-period profit-maximizing contract in period 1, and a single-period profit-maximizing contract in period 2. We solve by backwards induction and start with the second period.

As the second-period decision is analogous to the single-period short-term decision in section 3, the dominant supplier will offer either a two-part tariff, quantity discount or market-share discount, as all these contracts lead to maximum second-period profits. Hence, the dominant upstream firm earns $\pi_{M2}(c_2) = \pi^I_2(c_2) - \pi^o_{R2}(c_2)$ in the second period whilst the downstream firm gets its outside option $\pi^o_{R2}(c_2)$, both depending on c_2.

Solving for the optimal decision in the first period, all firms take into account the second-period outcome in their optimization. As the marginal cost c_2 depends on the first-period quantity q_{C1} sold by the competitive fringe, the dominant supplier as well as the downstream firm maximize long-run profits to solve for their optimal quantity, and contract decisions. Long-run profits of firm J, $J = M, R$ are defined by the present value

$$\Pi_J = \pi_{J1} + \delta \pi_{J2},$$

[15] That is, even if $c_1 > c$, the fringe's marginal cost in $t = 2$ might be larger or lower than M's cost c which means that the competitive fringe can become more efficient than the dominant supplier.

[16] Due to the considered inverse demand system, the optimal first-period purchase level q_{C1} of the fringe's good never exceeds the 'outside option' level q^o_{C1}.

[17] Section 5.2 shows that the qualitative results characterized in the previous section also hold for the case where the dominant supplier faces learning effects.

where single-period profits are defined by π_{Jt} $(t = 1, 2)$, and δ represents the (time-)discount factor.

Note that introducing learning-by-doing influences the quantity decision of the downstream firm as well as the contract decision of the dominant supplier. First, as the downstream firm's second period profits are given by its outside option $\pi^o_{R2}(c_2)$, this firm prefers larger learning effects of the competitive fringe, because learning effects decrease the fringe's cost. Hence the downstream firm's costs also decrease, as the fringe's wholesale price decreases. Therefore, the more learning decreases the fringe's cost, the larger the increase of downstream profits. In particular, with the help of the Envelope Theorem, the extent of this effect is given by

$$\frac{\partial \pi^o_{R2}}{\partial q_{C1}} = -\lambda \frac{\partial \pi^o_{R2}}{\partial c_2} = \lambda q^o_{C2} > 0,$$

where q^o_{C2} is the competitive fringe's second-period sales in the (long-run) outside option.

Second, the dominant upstream firm's second-period profits decrease when learning-by-doing increases (that is, when q_{C1} increases). In particular, this effect is given by

$$\frac{\partial \pi_{M2}}{\partial q_{C1}} = -\lambda \cdot \left(\frac{\partial \pi^I_2}{\partial c_2} - \frac{\partial \pi^o_{R2}}{\partial c_2}\right) = -\lambda \cdot (q^o_{C2} - q^I_{C2}).$$

As the joint-profit maximizing sales level q^I_{C2} of the competitive fringe is smaller than q^o_{C2} (by construction of the demand system)[18], the derivative $\frac{\partial \pi_{M2}}{\partial q_{C1}}$ is negative. Hence, the larger the fringe's learning-by-doing, the larger the competitive threat to the dominant supplier.

Together, there occur two opposing effects for second period outcomes: First, the downstream firm supports learning-by-doing by purchasing more of the competitive fringe's good. Second, the dominant supplier tends to restrict learning as it presents a competitive threat and decrease upstream profits in the first glance.

In the following, we analyze specific decisions in the first period. We start with the case where the dominant upstream firm offers a two-part tariff in period 1. Then, we analyze the optimal decision if the dominant upstream firm offers a quantity discount, or if it offers a market-share discount in period 1. We then compare these results to derive the optimal contract choice and the related outcome.

To ensure that long-run profits have a unique maximum in this context, we make the following, additional assumption.

Assumption 2.
Long-run joint profits are given by

$$\Pi_I(q_{M1}, q_{C1}) = (P_M(q_{M1}, q_{C1}) - c)q_{M1} + (P_C(q_{M1}, q_{C1}) - c_1)q_{C1} + \delta \pi^I_2(c_2(q_{C1})),$$

long-run downstream profits are given by $\Pi_R(q_{M1}, q_{C1}) - F_1$*, where*

$$\Pi_R(q_{M1}, q_{C1}) = (P_M(q_{M1}, q_{C1}) - w_{M1})q_{M1} + (P_C(q_{M1}, q_{C1}) - c_1)q_{C1} + \delta \pi^o_{R2}(c_2(q_{C1}))$$

according to the pricing structure, respectively the wholesale price w_{M1}*. Both are assumed to be concave. That is, the Hessian matrix of* Π_I *and* Π_R *is negative definite.*

[18] A proof of this relation is given in Section A.1.

Note that the downstream firm's net long-run profits $\Pi_R(q_{M1}, q_{C1})$ imply the optimal outcome for the second period. That is, the second-period contract is already included in the calculations. Second-period prices, especially the fixed fee F_2 are already payed and are not included in these long-run profits.

4.1.1 Two-part tariffs

We consider first that the dominant upstream supplier offers a simple two-part tariff in period 1. In the second period, profits are given by $\pi_{M2}(c_2)$ for the dominant supplier, and $\pi^o_{R2}(c_2)$ for the downstream firm, independent of the contract structure in period 2. Without loss of generality, we suppose here that the dominant supplier offers a two-part tariff in each period.

Solving by backwards induction, we start with the downstream sector. The downstream firm R accepts the two-part tariff in period 1 only if related long-run profits exceed the long-run outside option, which is 'purchasing only from the competitive fringe'[19], given by

$$\Pi^o_R = \max_{q_{C1}}(P_C(0, q_{C1}) - c_1)q_{C1} + \delta\pi^o_{R2}(c_2(q_{C1})). \tag{3}$$

When R accepts the two-part tariff, it maximizes net long-run profits

$$\Pi_R(q_{M1}, q_{C1}) = (P_M(q_{M1}, q_{C1}) - w^{2\text{PT}}_{M1})q_{M1} + (P_C(q_{M1}, q_{C1}) - c_1)q_{C1} + \delta\pi^o_{R2}(c_2(q_{C1})) \tag{4}$$

with respect to both quantities.[20] There are no additional constraints, thus both optimal quantity levels $q_{M1}(w_{M1})$, $q_{C1}(w_{M1})$ depend on the wholesale price w_{M1}, and are characterized by the first order conditions

$$\frac{\partial\Pi_R}{\partial q_{M1}} = \frac{\partial P_M}{\partial q_{M1}}q_{M1} + \frac{\partial P_C}{\partial q_{M1}}q_{C1} + P_M(q_{M1}, q_{C1}) - w_{M1} = 0, \tag{5}$$

$$\frac{\partial\Pi_R}{\partial q_{C1}} = \frac{\partial P_M}{\partial q_{C1}}q_{M1} + \frac{\partial P_C}{\partial q_{C1}}q_{C1} + P_C(q_{M1}, q_{C1}) - c_1 - \delta\lambda\frac{\partial\pi^o_{R2}}{\partial c_2} = 0. \tag{6}$$

Differentiating (5), (6) with respect to w_{M1}, and regarding the second-order conditions shows that an increase in w_{M1} decreases q_{M1}, but increases q_{C1}.[21] This relation is caused by the assumption of imperfect substitutes. When the wholesale price of the dominant supplier's good increases, the downstream firm would purchase less of this good and substitute with the other. Hence, R would purchase more of the fringe's good instead.

The dominant upstream firm decides about the profit-maximizing wholesale price and fixed fee, anticipating this quantity choice as well as the participation constraint of the downstream firm. That is, the dominant supplier maximizes its long-run profits Π_M subject to the participation constraint $\Pi_R(q_{M1}(w_{M1}), q_{C1}(w_{M1})) - F_1 \geq \Pi^o_R$.[22] Here, M's long-run profits are given by

$$\Pi_M = q_{M1}(w_{M1})(w_{M1} - c) + F_1 + \delta(\pi^I_2(c_2) - \pi^o_{R2}(c_2)).$$

[19]Furthermore, Π^o_R also represents downstream profits, when R decides to purchase only the competitive fringe's good in period 1, and purchases both goods in period 2 - independent of the second-period contract.

[20]The quantity choice of q_{C1} influences the second-period outcome. Therefore, long-run profits will be maximized which differs from the single-period profit maximization.

[21]The derivatives $\frac{\partial q_{M1}}{\partial w_{M1}}$ (< 0) and $\frac{\partial q_{C1}}{\partial w_{M1}}$ (> 0) can be found in Section A.1.

[22]As upstream profits are larger when M sells its good in both periods, it will always offer contract terms that achieve R's acceptance in both periods.

These long-run profits include the optimal second-period profits depending on c_2, and therefore on $q_{C1}(w_{M1})$, and first-period profits which depend on w_{M1} and F_1. Moreover, the first-period profits consists of two parts, 'variable profits' which are given by the transaction-based fees and 'fixed profits' based on the fixed fee.

As the participation constraint is binding in equilibrium, the fixed fee shifts all additional rents upwards, and the wholesale price is used to maximize (combined) long-run profits. The optimal contract terms are given as follows.

Lemma 3 (Two-part tariffs)**.**
When the dominant upstream firm offers short-run two-part tariffs, the profitable contracts are defined by

- w_{M1}^{2PT} *given by* $w_{M1}^{2PT} = c + \delta\lambda \frac{\partial \pi_{M2}}{\partial c_2} \frac{\partial q_{C1}/\partial w_{M1}}{\partial q_{M1}/\partial w_{M1}}$
  and $F_1^{2PT} = max_{q_{M1},q_{C1}} \Pi_R(q_{M1}, q_{C1}) - \Pi_R^o$*, in period 1,*
- $w_{M2}^{2PT} = c$ *and* $F_2^{2PT} = \pi_2^I(c_2^{2PT}) - \pi_{R2}^o(c_2^{2PT})$*, in period 2.*

Proofs are delegated to Section A.1.
The profit-maximizing outcome is characterized by the quantity levels q_{M1}^{2PT}, q_{C1}^{2PT}, which are given by

$$\frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} q_{C1} + P_M(q_{M1}, q_{C1}) - c - \delta\lambda \frac{\partial \pi_{M2}}{\partial c_2} \frac{\partial q_{C1}/\partial w_{M1}}{\partial q_{M1}/\partial w_{M1}} = 0,$$
$$\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} q_{C1} + P_C(q_{M1}, q_{C1}) - c_1 - \delta\lambda \frac{\partial \pi_{R2}^o}{\partial c_2} = 0,$$

as well as q_{M2}^I, q_{C2}^I, given by the second-period joint-profit maximizing levels.
As $\frac{\partial q_{C1}/\partial w_{M1}}{\partial q_{M1}/\partial w_{M1}} < 0$ and $\frac{\partial \pi_{M2}}{\partial c_2} > 0$ for all c_2, the optimal wholesale price of the first-period two-part tariff is smaller than marginal cost c. That is, in contrast to the benchmark situation, the dominant upstream firm uses below-cost pricing in the first period, to maximize long-run profits. The intuition for this pricing strategy lies in the assumptions of imperfect substitutes and complete information[23]. Suppose for a moment that the dominant supplier set the first-period wholesale price equal to marginal cost c, analog to the benchmark case. Then, the downstream firm would have chosen quantities $q_{M1}(c)$, $q_{C1}(c)$ according to the following first-order conditions

$$\frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} q_{C1} + P_M(q_{M1}, q_{C1}) - c = 0,$$
$$\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} q_{C1} + P_C(q_{M1}, q_{C1}) - c_1 - \delta\lambda \frac{\partial \pi_{R2}^o}{\partial c_2} = 0.$$

These conditions show that the optimal quantity level q_{M1}^{2PT} would have been lower and the level q_{C1}^{2PT} larger than calculated in Lemma 3. Yet, as the (overall) maximum profit for the dominant supplier is given in case of maximum joint profits, the supplier prefers the

[23] Note that complete information refers to the downstream firm's knowledge about the fringe's learning-by-doing effects. When the downstream firm does not know about learning-by-doing, R would purchase less of the fringe's good.

quantity levels which are given by the following first-order conditions

$$\frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} q_{C1} + P_M(q_{M1}, q_{C1}) - c = 0, \tag{7}$$

$$\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} q_{C1} + P_C(q_{M1}, q_{C1}) - c_1 - \delta\lambda \frac{\partial \pi_2^I}{\partial c_2} = 0. \tag{8}$$

As $\frac{\partial \pi_{R2}^o}{\partial c_2} < \frac{\partial \pi_2^I}{\partial c_2} < 0$, the downstream firm would thus choose a sales level $q_{C1}(c)$ for $w = c$, which is larger than preferable from the dominant supplier's point of view. That is, the downstream firm would provoke a level of learning effects for the competitive fringe, which is larger than preferable for the dominant supplier. In addition, the sales level of $q_{M1}(c)$ would be smaller than the preferable level for the upstream firm. That is, the joint-profit maximizing result does not occur for a wholesale price equal to marginal cost. This difference to the benchmark case occurs because firms anticipate the influence of the fringe's learning on second-period profits. Hence, by decreasing the wholesale price below cost, the dominant upstream firm makes the downstream firm choose a lower quantity $q_{C1}(w_{M1}^{\text{2PT}})$ and larger level $q_{M1}(w_{M1}^{\text{2PT}})$ than in case of $w_{M1} = c$.

Below-cost pricing is thus used by the dominant supplier to reduce the competitor's sales and to approach the joint-profit maximizing outcome which maximizes upstream profits. However, the true joint-profit maximizing outcome cannot be reached. Decreasing the wholesale price w_{M1} below cost increases the long-run downstream profits and therefore increases the fixed fee F_1. Furthermore, M's second-period profits increase when q_{C1} decreases. Yet, as the margin $w_{M1} - c$ is negative and the quantity level q_{M1} increases when w_{M1} decreases, M's first-period variable profits are negative and decreasing. Therefore, the profit maximizing level of w_{M1} will be larger than the level which would determine the joint-profit maximizing sales levels. That is, the quantity level q_{C1}^{2PT} is larger than in case of maximum industry profits, and q_{M1}^{2PT} is smaller.

Proposition 4 (Learning effects and two-part tariffs).
In contrast to the benchmark case, the dominant supplier's profit-maximizing two-part tariff does not lead to the joint-profit maximizing outcome when learning-by-doing occurs. As the first-period quantity $q_{C1}^{2PT} = q_{C1}(w_{M1}^{2PT})$ sold by the competitive fringe is larger than the joint-profit maximizing level q_{C1}^I, the competitive fringe's marginal costs c_2^{2PT} are smaller in case of two-part tariffs, than in case of maximum joint profit.

The proof is delegated to the Appendix. The overcompensation of learning-by-doing which is initiated by the downstream firm's profit maximization, cannot be influenced to achieve the quantity levels q_{M1}^I and q_{C1}^I which are preferable from the dominant supplier's perspective. The profit-maximizing quantity level q_{C1}^{2PT} is larger than q_{C1}^I for $\lambda > 0$, because it is not profitable for the dominant supplier to charge a lower wholesale price than w_{M1}^{2PT}.

In the following, we derive the dominant upstream firm's profit-maximizing quantity discount and market-share discount in period 1, and analyze whether these discounts influence the fringe's learning effects.

4.1.2 Quantity discounts

We now consider the case, where the dominant supplier offers a quantity discount in period 1. Without loss of generality we suppose that the upstream firm offers a quantity discount in period 2 as well. Second period profits are given by $\pi_{M2}(c_2)$ and $\pi^o_{R2}(c_2)$.

A quantity discount in period 1 is defined by a quantity threshold q^*_{M1}, an un-discounted wholesale price, a discounted wholesale price and the fixed fee F^{Q}_1. When the downstream firm purchases less than q^*_{M1} units of the dominant supplier's good, it has to pay the un-discounted wholesale price w_1 per unit. If it purchases at least q^*_{M1} units of M's good, the wholesale price is lower, given by the discounted price $w_{\mathrm{Q}1}$ per unit. As the dominant supplier has no incentive to offer a discount scheme which is then rejected by the single buyer, the un-discounted wholesale price w_1 will be unattractively large. That is, accepting the contract and purchasing less than the quantity target q^*_{M1} leads to lower profits for the downstream firm, than the outside option Π^o_R. Therefore, the downstream firm only decides whether to accept the quantity discount purchasing exactly or more than q^*_{M1} units of the dominant supplier, or to reject the offer and earn Π^o_R. When R accepts the discount scheme and purchases more than q^*_{M1} units, it chooses the same quantity levels as in the case of two-part tariffs. Hence, maximizing upstream profits would lead to the same prices and outcome as the profit-maximizing two-part tariff, where $q^*_{M1} < q^{\mathrm{2PT}}_{M1}$, in this case. When, in contrast, the discount condition is binding (R purchases exactly q^*_{M1} units of M), the downstream firm chooses $q_{C1}(q^*_{M1})$ with respect to q^*_{M1}, according to

$$\frac{\partial \Pi_R}{\partial q_{C1}} = \frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} q_{C1} + P_C(q_{M1}, q_{C1}) - c_1 - \delta\lambda \frac{\partial \pi^o_{R2}}{\partial c_2} = 0. \tag{6}$$

Similar to the optimization in case of two-part tariffs, the downstream firm anticipates the fringe's learning effects when choosing the quantity level q_{C1}. The influence of learning is given by $\lambda \frac{\partial \pi^o_{R2}}{\partial c_2}$. In contrast to two-part tariffs, where both quantities depend on the wholesale price, the downstream firm chooses only q_{C1}, and only with respect to the fixed level q^*_{M1}. In particular, the quantity forcing effect of these discount schemes gives the dominant supplier a more direct influence on the quantity choice q_{M1} of the downstream firm, compared to two-part tariffs. The dominant supplier maximizes long-run profits

$$\Pi_M = q^*_{M1} \cdot (w_{\mathrm{Q}1} - c) + F_1 + \delta \pi_{M2}(c_2)$$

subject to the participation constraint $\Pi_R(q^*_{M1}, q_{C1}(q^*_{M1})) - F_1 \geq \Pi^o_R$. Solving the optimization problem by using the Kuhn-Tucker conditions leads to the following profit-maximizing quantity discount terms.

Lemma 5 (Quantity discounts)**.**
The profit-maximizing (binding) quantity discounts are given by

- $(w_{\mathrm{Q}1}, F^Q_1)$ *and* q^*_{M1} *equal to* q^{2PT}_{M1} *in period 1,*
- $(w_{\mathrm{Q}2}, F^Q_2)$ *and* $q^*_{M2} = q^I_{M2}$, *in period 2,*

where the tuples $(w_{\mathrm{Q}1}, F^Q_1)$ *and* $(w_{\mathrm{Q}2}, F^Q_2)$ *are defined by* $F^Q_1 = \Pi_R(q^{2PT}_{M1}, q^{2PT}_{C1}) - \Pi^o_R$ *and* $F^Q_2 = \pi_{R2}(q^{2PT}_{M2}, q^{2PT}_{C2}) - \pi^o_{R2}(c_2)$.[24]

[24] Here $\Pi_R(q^{\mathrm{2PT}}_{M1}, q^{\mathrm{2PT}}_{C1})$ depends on $w_{\mathrm{Q}1}$, and $\pi_{R2}(q^{\mathrm{2PT}}_{M2}, q^{\mathrm{2PT}}_{C2})$ depends on $w_{\mathrm{Q}2}$.

Proofs can be found in Section A.1.

The profit-maximizing outcome is characterized by the quantity levels q_{M1}^{2PT}, q_{C1}^{2PT}, and $q_{M2}^{I}(c_2^{2PT})$, $q_{C2}^{I}(c_2^{2PT})$. Hence, both the binding as well as non-binding discount condition lead to the same result as a simple two-part tariff.

In both cases (binding, non-binding), the discount condition causes an over-specification of contract terms. In comparison to two-part tariffs, the dominant supplier faces an additional instrument, namely q_{M1}^{*}, when it offers a quantity discount. However, in case of both the binding and non-binding quantity discount, this additional instrument characterizes an over-specification of contract terms. For the non-binding condition, this is because q_{M1}^{*} has no influence on the downstream firm's optimization. For the binding discount condition, q_{M1}^{*} serves as a control variable, but it is the only one to influence the downstream quantity choice q_{C1}. Hence, the discounted wholesale price as well as the fixed fee achieve R's acceptance, but have no influence on the quantity choice of the downstream firm. Defining a discounted price w_{Q1} as well as a fee F_1^{Q} is therefore an over-specification of contract terms. Setting no fixed fee, and choosing the related discounted wholesale price according to $\Pi_R(q_{M1}^{\mathrm{2PT}}, q_{C1}^{\mathrm{2PT}}) = \Pi_R^o$ (where $q_{M1}^{*} = q_{M1}^{\mathrm{2PT}}$) is for example one specific, profit-maximizing contract.[25]

Proposition 6 (Learning effects and quantity discounts)**.**
As profit-maximizing quantity discounts lead to the same outcome as two-part tariffs, the competitive fringe's marginal costs are c_2^{2PT}. Hence, the degree of C's learning-by-doing is the same as in case of profit-maximizing two-part tariffs.

When the dominant supplier offers quantity discounts (as one type of conditional discounts), these do not restrict the competitive fringe's learning-by-doing. Instead, this form of conditional discounts leads to the same result that is generated without a discount scheme.

However, quantity discounts induce an advantage over two-part tariffs, as they can specify a discounted wholesale price above marginal cost c (and a related fixed fee). By offering such a contract, the dominant supplier earns the same profit as in case of two-part tariffs, without using below-cost pricing.[26]

4.1.3 Market-share discounts

A market-share discount in period 1 defines a share threshold ρ_1^{*}, the discounted wholesale price w_{MS1}, an un-discounted price w_1 as well as a fixed fee F_1^{MS}. Compared to quantity discounts, market-share discounts have no direct influence on absolute quantity levels, but relative levels. In the following we suppose that the dominant supplier offers a market-share discount in period 1, and, without loss of generality, in period 2 as well.

In the first period, the downstream firm R decides whether to accept the contract, and optionally the discount, or reject the offer. As the dominant firm induces R to fulfill the

[25] A second combination is $w_{\mathrm{Q1}} = 0$ and $F_1^{\mathrm{Q}} = \Pi_R(q_{M1}^{\mathrm{2PT}}, q_{C1}^{\mathrm{2PT}})|_{w_{Q1}=0} - \Pi_R^o$.

[26] As antitrust authorities claim that below-cost pricing is anticompetitive, quantity discounts may characterize an alternative pricing scheme.

discount condition, the un-discounted wholesale price is relatively large, leading to (long-run) downstream profits below Π_R^o. Maximizing profits $\Pi_R(q_{M1}, \frac{1-\rho_1}{\rho_1} q_{M1})$ subject to the discount condition $\rho_1 \geq \rho_1^*$, the downstream firm can decide to purchase more or exactly at the threshold. When the downstream firm prefers to purchase a larger share ρ_1^* of the dominant firm's product, the optimization problem is similar to the case of two-part tariffs.[27] Yet, in case of a binding discount condition, the downstream firm purchases exactly at the share threshold ρ_1^*. Accepting M's contract, this means that the firm maximizes profits only with respect to aggregate purchase while taking ρ_1^* as given. The first-order condition representing optimal purchase in case of a binding discount is given by

$$\begin{aligned}\frac{\partial \Pi_R}{\partial q_{M1}} &= \frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} \frac{1-\rho_1}{\rho_1} q_{M1} + P_M(q_{M1}, \frac{1-\rho_1}{\rho_1} q_{M1}) - w_{\text{MS1}} \\ &+ \frac{1-\rho_1}{\rho_1} \left(\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} \frac{1-\rho_1}{\rho_1} q_{M1} + P_C(q_{M1}, \frac{1-\rho_1}{\rho_1} q_{M1}) - c_1 - \delta\lambda \frac{\partial \pi_{R2}^o}{\partial c_2} \right) = 0. \qquad (9)\end{aligned}$$

The implicit function theorem shows that quantity q_{M1} decreases, when the wholesale price w_{MS1} increases. As the quantity sold by the competitive fringe equals $q_{C1} = \frac{1-\rho_1^*}{\rho_1^*} q_{M1}$, this quantity level also decreases when w_{MS1} increases. Compared to the previous contracts, the binding market-share discount hinders the downstream firm to substitute goods when the wholesale price increases. Instead, this firm would lower both quantity levels to the same, proportional extent.

Furthermore, equation (9) shows that the purchase level q_{M1} depends not only on the discounted wholesale price w_{MS1}, but also on the share ρ_1^*. For that reason the dominant upstream supplier possesses two instruments to control for q_{M1} and $q_{C1}(= \frac{1-\rho_1^*}{\rho_1^*} q_{M1})$, when it maximizes its profits Π_M. Namely, ρ_1^* and w_{MS1} control the quantity levels, and F_1^{MS} is used to shift rents upwards.

Lemma 7 (Market-share discounts)**.**
The profit maximizing binding market-share discounts are given by

- $\rho_1^* = \frac{q_{M1}^I}{q_{M1}^I + q_{C1}^I}$,
  w_{MS1} *given by* $w_{MS1} = c + \delta\lambda \frac{\partial \pi_{M2}}{\partial c_2} \frac{1-\rho_1^*}{\rho_1^*}$,
  $F_1^{MS} = (P_M(q_{M1}^I, q_{C1}^I) - w_{MS1}) q_{M1}^I + (P_C(q_{M1}^I, q_{C1}^I) - c_1) q_{C1}^I + \delta \pi_{R2}^o(c_2^I) - \Pi_R^o$

  in the first period, and

- $\rho_2^* = \frac{q_{M2}^I}{q_{M2}^I + q_{C2}^I}$,
  $w_{MS2} = c$,
  $F_2^{MS} = \pi_2^I(c_2^I) - \pi_{R2}^o(c_2^I)$

  in the second period.

Proofs are delegated to the Appendix, Section A.1.

[27] Considering that ρ_1^* is smaller than the preferred quantity levels, the downstream firm maximizes profits, i.e. with respect to both quantity levels, as before.

The first-order conditions of the dominant supplier's optimization problem are given by

$$w_{MS1} - c - \delta\lambda \frac{1-\rho_1^*}{\rho_1^*}\frac{\partial \pi_{M2}}{\partial c_2} = 0 \tag{10}$$

$$\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} \frac{1-\rho_1^*}{\rho_1^*} q_{M1} + P_C(q_{M1}, \frac{1-\rho_1^*}{\rho_1^*} q_{M1}) - c_1 - \delta\lambda \frac{\partial \pi_2^I}{\partial c_2} = 0, \tag{11}$$

see Section A.1. Inserting these conditions in the downstream firm's first-order condition (9) shows that the optimal quantity choice implicates the joint-profit maximizing levels q_{M1}^I, q_{C1}^I, as well as q_{M2}^I, q_{C2}^I.

Furthermore, as $\frac{\partial \pi_{M2}}{\partial c_2}$ is positive for all $c_2 \in [0, c_1]$, the discounted wholesale price is larger than marginal cost. This result stems from the fact that the binding market-share discount restricts the downstream firm's relative purchase levels: If the downstream firm increases its purchase of the fringe's good, it needs to increase the purchase of M's good, too. Suppose for a moment that the dominant upstream supplier would offer a discount condition specifying ρ_1^* (as given in the Lemma) and a wholesale price which equals marginal costs c, similar to the benchmark case. Inserting the dominant firm's first-order condition (11) as well as $w_{M1} = c$ into the downstream firm's decision, which is given by equation (9), characterizes the downstream firm's quantity choice, in this case. In comparison to the joint-profit maximizing levels, which are characterized by (7) and (8), however, the downstream firm's purchase levels are larger. The reason for this result is that the downstream firm anticipates the fringe's learning effects. Anticipating its second-period profits, the downstream firm purchases a larger level of q_{C1}, and due to the binding share threshold ρ_1^* a larger level of q_{M1}. Raising the wholesale price therefore decreases both quantity levels, leading to the joint-profit maximizing quantity as well as price levels which are preferable for the dominant upstream firm. Thus, the profit-maximizing market-share discounts yield the joint-profit maximizing outcome as the binding discount condition represents an additional control variable to maximize upstream profits.[28]

Proposition 8 (Learning-by-doing and market-share discounts).
The dominant supplier's profit-maximizing market-share discounts lead to the joint-profit maximizing result. The marginal cost level of the competitive fringe is therefore given by c_2^I, which is larger than the level for two-part tariffs, and quantity discounts, c_2^{2PT}.

Thus, the learning effect of the competitive fringe has an influence on the short-term contract decision of the dominant supplier: Compared to the benchmark case without learning-by-doing (where the dominant supplier chooses either a two-part tariff or a conditional discount scheme) market-share discounts are strictly more profitable for the dominant supplier, when learning occurs.

[28] In addition, note that the fixed fee F_1^{MS} can also be written as $F_1^{MS} = \Pi_I - \Pi_R^o - \delta\{\pi_{M2}(c_2^I) + \lambda \frac{1-\rho_1}{\rho_1} \frac{\partial \pi_{M2}}{\partial c_2} q_{M1}^I\}$. Since $\Pi_I - \Pi_R^o$ has to be positive (otherwise M would not offer this contract) and since the last subtrahend is positive as well, it is not clear whether the fee is positive or negative. The sign and size of F_1^{MS} depends on the substitutability of the products, and of the learning parameter λ.

Corollary 9 (Profit-maximizing short-term contracts).
When only short-term contracts are feasible, the dominant supplier will offer a binding market-share discount in period 1, combined with a two-part tariff, quantity discount or market-share discount in period 2.

This is because learning influences the decision of the downstream firm such that this monopolistic firm does not internalize industry profits. Instead, the downstream firm maximizes its long-run profits where it prefers to purchase more from the learning competitive fringe. A simple two-part tariff of the dominant supplier, defining a wholesale price and fixed fee, does not ensure enough instruments to control for the downstream quantity choice. Therefore, two-part tariffs cannot lead to maximum profits for the dominant supplier (given by the joint-profit maximizing outcome). Quantity discounts cannot improve the result as the additional discount condition characterize an over-specification for the already present variables, and no additional control. Binding market-share discounts, however, solve for the maximum joint profit. They specify an additional control variable by introducing the market-share discount condition.

Accordingly, a competitive threat given by a rival's learning-by-doing effects provides an additional explanation for the granting of market-share discounts by a dominant supplier.[29] The supplier uses market-share discounts to achieve the joint-profit maximizing outcome, which derive maximum upstream profits. It utilizes the fringe's learning effects as these increase the downstream firm's profits, and therefore the fixed fee which the dominant supplier will charge. The fringe's learning effects which are preferred by the dominant supplier, are characterized by c_2^I. That is, the dominant supplier has no incentive to exclude the competitive fringe. Yet, as two-part tariffs and quantity discounts lead to larger learning effects as $c_2^{2PT} < c_2^I$, the profitable contract choice of the dominant supplier restricts the fringe's learning effects to a certain extent.

4.2 Long-term contracts

In this section, we introduce long-run contracts in the context of learning-by-doing.

These contracts are often said to have anticompetitive effects. In particular, contracts that are set for a long time period, respectively several (short-term) periods, can have a larger binding effect on buyers. This implies that competitors have fewer possibilities to conclude profitable contracts with buyers.[30] We therefore examine the influence of the rival's learning-by-doing on the long-term contract choice.

Long-term two-part tariffs, quantity discounts and market-share discounts are characterized by the fact that all contractual terms are set in the initial period and cannot be renegotiated. In our setting, the dominant supplier defines wholesale prices for both, the first and second period, in its long-term contract, which is offered in the first period. Fur-

[29] Note however that this explanation does not necessarily have anti-competitive reasons.

[30] Faella (2008) notes on p.386: *"as the duration of the reference period and the time already elapsed increase, the pressure on buyers becomes more intense."*

thermore, the dominant supplier charges a fixed fee only in the initial period. We suppose that the supplier commits itself to these contractual terms.

If the dominant supplier sets wholesale prices equal to marginal costs, the downstream firm would maximize long-run industry profits. Thus, by charging the fixed fee $\Pi_I - \Pi_R^o$ in period 1, where Π_I defines maximum long-run industry profits, the dominant supplier M makes downstream firm R accept the long-run contract. By construction, the joint-profit maximizing outcome is reached and the dominant upstream firm earns maximum profits.

When the dominant supplier additionally grants discounts, the joint-profit maximizing result is also achieved, since the additional discount condition would be adapted to this profit maximizing result.

Proposition 10 (Long-run contracts)**.**
Long-run contracts, either with or without discount conditions, lead to the joint-profit maximizing outcome. The dominant firm will offer contracts that specify the fixed fee $F = \Pi_I - \Pi_R^o$ *and (discounted) wholesale prices* $w_{M1} = w_{M2} = c$. *In case of conditional discounts, the threshold levels are characterized by the joint-profit maximizing levels. Hence, M earns* $\Pi_M = \Pi_I - \Pi_R^o$ *and R earns* $\Pi_R = \Pi_R^o$, *in the long run.*

Similar to the benchmark case where learning-by-doing is banned, all contracts - two-part tariffs, quantity discounts and market-share discounts - lead to the same result which is optimal from M's point of view. The intuitive reason lies in the fixed fee which is now implemented in the first period. By setting the wholesale prices equal to marginal cost, the downstream firm is induced to maximize long-run joint profits. In particular, the downstream firm internalizes second-period industry profits, leading to the joint-profit-maximizing level of learning-by-doing.

Altogether, the dominant upstream firm prefers to offer long-run contracts (either with or without discounts) or short-run market-share discounts. All these contracts lead to maximum profits for the dominant supplier, and to the joint profit maximization. In contrast, short-run two-part tariffs as well as quantity discounts do not lead to maximum profits for the dominant supplier. In case of these contracts, the competitive fringe would sell more of its good and would therefore face larger learning effects, thus larger efficiency gains. For this reason, M's profit-maximizing contract choice restricts the competitor's learning effects.

5 Robustness and extensions

In the previous section, we showed that the extent of a competitive fringe's learning-by-doing is restricted by the dominant supplier's market-share discounts, compared to two-part tariffs or quantity discounts. In the following, we extend the model to network effects and analyze the case where both the dominant supplier as well as the competitive fringe face learning effects.

5.1 Network effects

As learning-by-doing affects the pricing decision of upstream firms (i.e. the competitive fringe's wholesale price), it also influences the downstream quantity decision over time. As such, learning-by-doing represents one type of inter-temporal externalities which may change the market structure. A second type of inter-temporal externalities are network effects which operate in a similar way.[31] In case of network effects, demand, or respectively the willingness-to-pay increases in dependence of the quantity already sold of the product.

In this subsection, we consider the case where the competitive fringe faces network effects instead of learning-by-doing. As will be shown, network effects lead to the same qualitative results as learning-by-doing.[32]

We suppose that both the dominant supplier as well as the competitive fringe face constant marginal cost over time. In the following setting, however, the inverse demand structure changes: In the first period, we consider that final consumers' demand is characterized by

$$P_M(q_{M1}, q_{C1}),$$
$$P_C(q_{M1}, q_{C1}).$$

In the second period, demand for the fringe's good increases proportionally to the sold quantity in period 1, q_{C1}. For simplicity, we define the second-period inverse demand system as follows:

$$P_M(q_{M2}, q_{C2}),$$
$$P_C(q_{M2}, q_{C2}) + \kappa q_{C1}.$$

That is, the price-cost margin for the competitive fringe's good (according to the downstream firm's optimization problem) is given by $P_C(q_{M2}, q_{C2}) + \kappa q_{C1} - c$. In case of learning-by-doing, this downstream mark-up was given by $P_C(q_{M2}, q_{C2}) - (c - \lambda q_{C1})$. Hence, the downstream profit maximization only differs in the parameter κ, compared to the case with learning-by-doing where λ was the key parameter.

That is, when the parameters κ and λ are equal, the downstream firm chooses the same quantities, with regard to the wholesale prices, as in case of learning-by-doing. Therefore, the optimal decision of the dominant supplier would also be similar to the previous setting and the same quantitative results would occur. In contrast, when κ differs from λ, however, the quantitative results differ. With regard to wholesale prices, the downstream firm purchases more from the fringe's good if κ is larger than λ, and vice versa. The dominant supplier's profit maximization reacts on this change in quantities, but derives the same

[31]For a definition of network and learning effects see for example Sutton (2001).

[32]Sutton (2001) shows that network effects operate in the same way as learning-by-doing effects when there are N initially identical firms which all face network/learning-by-doing effects. Yet, he identifies points of difference, as for example quantitative differences regarding the results. In our setting, we show that even in a vertical context where a competitive fringe face network effects while the dominant upstream firm does not, qualitative results are similar.

qualitative results as before. Hence, even though network effects might lead to different quantitative results, qualitative results are similar to the learning-by-doing setting.[33]

5.2 M's learning-by-doing and network effects

In Section 4, we assumed that the dominant upstream firm faces neither learning-by-doing nor network effects - in contrast to the learning competitive fringe. The general idea was to focus on the impact of discounts on the competitive fringe's learning or network effects. However, the dominant firm typically has a first-mover advantage. Therefore, restricting the dominant supplier in its growth seems arbitrary.

In this Section, we extend the model to allow for learning-by-doing by the dominant upstream firm.[34] In that context, allowing for these effects by the dominant supplier does not change any of the results: Short term market-share discounts and long-run contracts maximize the dominant supplier's profit. Yet, short-term quantity discounts and simple two-part tariffs lead to more learning for the competitive fringe.

This is because the downstream firm internalizes only the fringe's learning and network effects, when the dominant supplier offers short-term contracts. In this case, the dominant supplier's cost (or demand parameter) does not affect the downstream firm's second-period profit. Hence, the downstream firm anticipates C's inter-temporal effects, but does not consider M's effects.

6 Conclusion

In this paper, we examine the contract choice of a dominant upstream firm, facing a competitive threat caused by a competitive fringe's learning-by-doing effects. We consider a two-period model in which the dominant supplier offers either short-term contracts (specifying contract terms only for a single period) or long-term contracts (specifying contract terms for all periods in the first stage) to a single downstream firm. The contract structure is either a simple two-part tariff, a quantity discount or market-share discount.

We find that particularly in case of short-term contracts, the fringe's learning effect has an impact on the dominant supplier's contract choice: While all considered contract structures can derive the joint-profit maximizing result (which is the dominant supplier's profit maximizing result) when learning does not occur, short-term two-part tariffs and quantity discounts cannot lead to the joint-profit maximizing result when the competitive fringe faces learning effects. The reason for this result stems from the downstream firm's learning-supporting quantity choice: The dominant supplier's quantity discounts as well as two-part tariffs do not provide enough instruments to control for the downstream quantity choice. Furthermore, two-part tariffs and quantity discounts lead to the same profits for

[33]Note that κ is not restricted by the fringe's marginal cost. That is, $c_1 - \kappa q_{C1}$ can be negative as well. Hence, it is possible that the downstream firm prefers to purchase zero units of the dominant supplier's good.

[34]As before, network effects have the same qualitative effects.

the dominant supplier. That is, the additional discount offer conditioned on a quantity threshold does not affect the final result. Quantity discounts that do not define fixed fees have the same effect as two-part tariffs. Therefore, the additional fixed fee has no influence on the downstream firm's purchasing decision and hence characterizes an over-specification of contract terms.

Short-term market-share discounts characterize the best contract choice for the dominant supplier, when learning-by-doing occurs. This is because the discount condition characterizes an additional control variable for the dominant supplier. That is, binding market-share discounts, combined with a fixed fee, restrict the supporting effect of the downstream firm and lead to maximum profits for the dominant supplier.

In contrast to these short-term contracts, all considered long-term contracts lead to maximum profits for the dominant supplier, given by the joint-profit maximizing result. As this result is also given in the benchmark situation without learning-by-doing, the competitive fringe's learning effects have no influence on the dominant supplier's contract choice, when learning occurs.

Moreover, in comparison to these long-term contracts and short-term market-share discounts, short-term two-part tariffs and quantity discounts yield larger learning effects, hence a larger decrease of marginal cost, for the competitive fringe's good. As the dominant supplier's two-part tariffs cannot completely restrict the downstream firm's purchase of the fringe's good, the fringe's cost decrease more than in case of joint-profit maximization. Therefore, the profitable contract choice of the dominant supplier does actually limit learning-by-doing, as standard contracts without discount conditions lead to larger learning effects. Similar results are achieved for network effects.

This paper contributes to the literature by analyzing discount schemes in a dynamic context where rivals' learning-by-doing effects generate a growing competitive threat for a dominant supplier. By comparing different discount schemes in a setting which is driven by learning-by-doing effects, this paper presents a novel explanation for the use of market-share discounts and shows that these discounts can hinder competitors' efficiency gains.

In a next step, it would be interesting to prove whether the assumption of an unstrategic upstream competitor is important to achieve our qualitative results. In contrast to the competitive fringe, a strategic upstream competitor could offer its own discount terms which may influence the dominant supplier's decision. In particular, the timing seems to play an important role in this context. This is left for future research.

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A Appendix

A.1 Learning-by-doing, optimal contract offers

For ease of comprehension of the proofs and calculations relating to the propositions, we firstly derive the optimal/profit-maximizing long-run outside option and joint profit.

Outside option:

The downstream firm's outside option is defined by purchasing only from the competitive fringe C. The optimal profit in the second period is given by

$$\pi^o_{R2}(c_2) = max_{q_{C2}} \ (P_C(0, q_{C2}) - c_2)q_{C2}.$$

The first-order condition characterizing the optimal quantity q^o_{C2} is given by

$$\frac{\partial P_C(0, q_{C2})}{\partial q_{C2}} q_{C2} + P_C(0, q_{C2}) - c_2 = 0. \tag{12}$$

The second order condition is negative, by definition, characterizing the unique maximum. In the first period, the downstream firm chooses q^o_{C1} in case of purchasing only from C, given by

$$q^o_{C1} = argmax_{q_{C1}} \ (P_C(0, q_{C1}) - c_1)q_{C1} + \delta\pi^o_{R2}(c_2).$$

The related profits are $\Pi^o_R = (P_C(0, q^o_{C1}) - c_1)q^o_{C1} + \delta\pi^o_{R2}(c^o_2)$ in the long run.

Joint profit:

Second period (solving by backwards induction):

$$\pi_{I2} = (P_M(q_{M2}, q_{C2}) - c)q_{M2} + (P_C(q_{M2}, q_{C2}) - c_2)q_{C2}.$$

Optimal second-period quantities $q^I_{M2}(c_2)$, $q^I_{C2}(c_2)$ are considered to be positive and given by

$$\frac{\partial P_M}{\partial q_{M2}} q_{M2} + \frac{\partial P_C}{\partial q_{M2}} q_{C2} + P_M(q_{M2}, q_{C2}) - c = 0, \tag{13}$$

$$\frac{\partial P_M}{\partial q_{C2}} q_{M2} + \frac{\partial P_C}{\partial q_{C2}} q_{C2} + P_C(q_{M2}, q_{C2}) - c_2 = 0. \tag{14}$$

As a two-part tariff, quantity discount, and market-share discount yield joint-profit maximization in a single period, these first-order conditions characterize the optimal second-period outcome in all these cases, leading to profits $\pi_{M2}(c_2) = \pi^I_2(c_2) - \pi^o_{R2}(c_2)$, where $\pi^I_2(c_2)$ represents maximum joint profit for the single period 2, and $\pi^o_{R2}(c_2)$ is the outside option for period 2.

First period:

Maximizing joint profit Π_I in the first period, we differentiate with respect to quantities q_{M1}, q_{C1}.

$$\Pi_I = (P_M(q_{M1}, q_{C1}) - c)q_{M1} + (P_C(q_{M1}, q_{C1}) - c_1)q_{C1} + \delta\pi_{I2}(c_2)$$

The first-order conditions which characterize the joint-profit maximizing quantities are given by (13), (14) and

$$\frac{\partial \Pi_I}{\partial q_{M1}} = \frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} q_{C1} + P_M(q_{M1}, q_{C1}) - c = 0 \tag{15}$$

$$\frac{\partial \Pi_I}{\partial q_{C1}} = \frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} q_{C1} + P_C(q_{M1}, q_{C1}) - c_1 - \delta\lambda \frac{\partial \pi_{I2}}{\partial c_2} = 0 \tag{16}$$

Following assumption 2, the Hessian matrix of Π_I is negative definite. That is, the leading principal minors are $\frac{\partial^2 \Pi_I}{\partial q_{M1}^2} < 0$, and $\frac{\partial^2 \Pi_I}{\partial q_{M1}^2} \frac{\partial^2 \Pi_I}{\partial q_{C1}^2} - \frac{\partial^2 \Pi_I}{\partial q_{M1} \partial q_{C1}} \frac{\partial^2 \Pi_I}{\partial q_{C1} \partial q_{M1}} > 0$. Note that in this case: $\frac{\partial^2 \Pi_I}{\partial q_{C1}^2} < 0$, and $\frac{\partial^2 \Pi_I}{\partial q_{C1} \partial q_{M1}} = \frac{\partial^2 \Pi_I}{\partial q_{M1} \partial q_{C1}} < 0$, due to the assumptions on the inverse demand system.
Furthermore, it follows that $\frac{\partial^2 \Pi_I}{\partial q_{C1}^2} = \frac{\partial^2 P_M}{\partial q_{C1}^2} q_{M1} + \frac{\partial^2 P_C}{\partial q_{C1}^2} q_{C1} + 2\frac{\partial P_C}{\partial q_{C1}} + \delta\lambda^2 \frac{\partial^2 \pi_2^I}{\partial c_2^2} < 0$, where $\frac{\partial^2 \pi_2^I}{\partial c_2^2} = -\frac{\partial q_{C2}^I}{\partial c_2} =$

$$= -\frac{\frac{\partial^2 P_M}{\partial q_{M2}^2} q_{M2} + \frac{\partial^2 P_C}{\partial q_{M2}^2} q_{C2} + 2\frac{\partial P_M}{\partial q_{M2}}}{(\frac{\partial^2 P_M}{\partial q_{M2}^2} q_{M2} + \frac{\partial^2 P_C}{\partial q_{M2}^2} q_{C2} + 2\frac{\partial P_M}{\partial q_{M2}})(\frac{\partial^2 P_M}{\partial q_{C2}^2} q_{M2} + \frac{\partial^2 P_C}{\partial q_{C2}^2} q_{C2} + 2\frac{\partial P_C}{\partial q_{C2}}) - (\frac{\partial^2 P_M}{\partial q_{M2} \partial q_{C2}} q_{M2} + \frac{\partial^2 P_C}{\partial q_{M2} \partial q_{C2}} q_{C2} + \frac{\partial P_M}{\partial q_{C2}} + \frac{\partial P_C}{\partial q_{M2}})^2} > 0$$

(by using the implicit function theorem for the second-period first-order conditions).

Note that $q_{C2}^I(c_2) < q_{C2}^o(c_2)$:
Per definition of the inverse demand structure:
$P_C(q_{M2}, q_{C2}) > P_C(0, q_{C2})$ for $q_{M2} > 0$,
and $\frac{\partial P_M}{\partial q_{C2}} < 0$, $\frac{\partial P_C(q_{M2}, q_{C2})}{\partial q_{C2}} \leq \frac{\partial P_C(0, q_{C2})}{\partial q_{C2}}$.
Hence, for all $q_{M2} > 0$, the left-hand side (LHS) of equation (12) is larger than the LHS of equation (14). Thus, the quantity level $q_{C2}^o(c_2)$ is larger than $q_{C2}^I(c_2)$ (for the optimal level of $q_{M2}^I(c_2) > 0$).

Proof of Lemma 3:
In this case, the dominant upstream firm offers a two-part tariff in both periods. We solve for the profit-maximizing contract by backwards induction. In the second period, (13), (14) characterize the optimal outcome, and lead to profits $\pi_{M2}(c_2)$ for M, and $\pi_{R2}^o(c_2)$ for R.

Downstream:
In the first period, the downstream firm maximizes profits

$$\Pi_R(q_{M1}, q_{C1}) = (P_M(q_{M1}, q_{C1}) - w_{M1}) q_{M1} + (P_C(q_{M1}, q_{C1}) - c_1) q_{C1} - F_1 + \delta \pi_{R2}^o(c_2)$$

with respect to both quantities q_{M1}, q_{C1}. The following first-order conditions characterize the optimal choice $q_{M1}(w_{M1})$, $q_{C1}(w_{M1})$ in dependance of the wholesale price:

$$\frac{\partial \Pi_R}{\partial q_{M1}} = \frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} q_{C1} + P_M(q_{M1}, q_{C1}) - w_{M1} = 0 \tag{17}$$

$$\frac{\partial \Pi_R}{\partial q_{C1}} = \frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} q_{C1} + P_C(q_{M1}, q_{C1}) - c_1 - \delta\lambda \frac{\partial \pi_{R2}^o}{\partial c_2} = 0 \tag{18}$$

Considering that the Hessian matrix of Π_R is negative definite ensures that $q_{M1}(w_{M1})$ and $q_{C1}(w_{M1})$ represent the maximum of Π_R.

Note that $\frac{\partial q_{M1}}{\partial w_{M1}} < 0$, $\frac{\partial q_{C1}}{\partial w_{M1}} > 0$:
By differentiating (17), (18) with respect to w_{M1} and regarding the second-order conditions (/implicit function theorem), we get
$\frac{\partial q_{M1}}{\partial w_{M1}} = -\frac{1}{a}(\frac{\partial^2 P_M}{\partial q_{C1}^2} q_{M1} + \frac{\partial^2 P_C}{\partial q_{C1}^2} q_{C1} + 2\frac{\partial P_C}{\partial q_{C1}} + \delta\lambda^2 \frac{\partial^2 \pi^o_{R2}}{\partial c_2^2}) < 0,$
$\frac{\partial q_{C1}}{\partial w_{M1}} = -\frac{1}{a}(\frac{\partial^2 P_M}{\partial q_{C1}\partial q_{M1}} q_{M1} + \frac{\partial^2 P_C}{\partial q_{C1}\partial q_{M1}} q_{C1} + \frac{\partial P_C}{\partial q_{M1}} + \frac{\partial P_M}{\partial q_{C1}}) > 0$
where a is the (second) leading principal minor of the Hessian matrix of Π_R: $a = a_{11}a_{22} - a_{12}a_{21}$, with
$a_{11} = (\frac{\partial^2 P_M}{\partial q_{M1}^2} q_{M1} + \frac{\partial^2 P_C}{\partial q_{M1}^2} q_{C1} + 2\frac{\partial P_M}{\partial q_{M1}})$
$a_{22} = (\frac{\partial^2 P_M}{\partial q_{C1}^2} q_{M1} + \frac{\partial^2 P_C}{\partial q_{C1}^2} q_{C1} + 2\frac{\partial P_C}{\partial q_{C1}} + \delta\lambda^2 \frac{\partial^2 \pi^o_{R2}}{\partial c_2^2})$
$a_{12} = a_{21} = (\frac{\partial^2 P_M}{\partial q_{C1}\partial q_{M1}} q_{M1} + \frac{\partial^2 P_C}{\partial q_{C1}\partial q_{M1}} q_{C1} + \frac{\partial P_C}{\partial q_{M1}} + \frac{\partial P_M}{\partial q_{C1}})$. Moreover,

$$\frac{\frac{\partial q_{C1}}{\partial w_{M1}}}{\frac{\partial q_{M1}}{\partial w_{M1}}} = -\frac{a_{12}}{a_{22}} = -\frac{\frac{\partial^2 P_M}{\partial q_{C1}\partial q_{M1}} q_{M1} + \frac{\partial^2 P_C}{\partial q_{C1}\partial q_{M1}} q_{C1} + \frac{\partial P_C}{\partial q_{M1}} + \frac{\partial P_M}{\partial q_{C1}}}{\frac{\partial^2 P_M}{\partial q_{C1}^2} q_{M1} + \frac{\partial^2 P_C}{\partial q_{C1}^2} q_{C1} + 2\frac{\partial P_C}{\partial q_{C1}} + \delta\lambda^2 \frac{\partial^2 \pi^o_{R2}}{\partial c_2^2}} \tag{19}$$

where the denominator is negative, by definition. (This is due to the negative definite Hessian matrix.)

Upstream:
M's profits are given by

$$\Pi_M = q_{M1}(w_{M1} - c) + F_1 + \delta(\pi_2^I(c_2) - \pi^o_{R2}(c_2)).$$

We optimize with respect to w_{M1}, F_1 and subject to the participation constraint

$$\Pi_R(q_{M1}(w_{M1}), q_{C1}(w_{M1})) - F_1 \geq \Pi_R^o.$$

The participation constraint is binding, which leads to the following simplified optimization problem:

$$\begin{aligned} &\max_{w_{M1}} (P_M(q_{M1}(w_{M1}), q_{C1}(w_{M1})) - c) q_{M1}(w_{M1}) \\ &\qquad + (P_C(q_{M1}(w_{M1}), q_{C1}(w_{M1})) - c_1) q_{C1}(w_{M1}) + \delta\pi_2^I(c_2) - \Pi_R^o. \end{aligned}$$

Therefore, the optimization depends on w_{M1}, in the first instance. The difference between the joint profit function and the objective function of M's optimization problem is given by the quantities q_{M1}, q_{C1} which depend on w_{M1}.

Using (17), (18), and differentiating the simplified objective function, we get the optimal wholesale price $w_{M1}^{2\text{PT}}$ by

$$(w_{M1}^{2\text{PT}} - c)\frac{\partial q_{M1}}{\partial w_{M1}} - \delta\lambda \frac{\partial \pi_{M2}}{\partial c_2}\frac{\partial q_{C1}}{\partial w_{M1}} = 0 \qquad (*)$$

Since M's second-period profit increases with respect to c_2, for all $c_2 \in [0, c_1]$, the optimal wholesale price from M's point of view is smaller than marginal cost.[35]

[35] This wholesale price characterizes the maximum because the second order condition is negative.

Moreover, inserting the optimal wholesale price in (17), the optimal outcome in case of two-part tariffs is given by (13), (14), (18) and (20) (with regard to $q_{M1}(w_{M1})$, $q_{C1}(w_{M1})$ characterized by (17), (18)):

$$\frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} q_{C1} + P_M(q_{M1}, q_{C1}) - c - \delta\lambda \frac{\partial \pi_{M2}}{\partial c_2} (\frac{\partial q_{C1}/\partial w_{M1}}{\partial q_{M1}/\partial w_{M1}}) = 0 \tag{20}$$

The profit maximizing first-period two-part tariff is hence given by $(w_{M1}^{\text{2PT}}, F_1^{\text{2PT}})$ where (*) characterizes the wholesale price, and $F_1^{\text{2PT}} = max_{q_{M1}, q_{C1}}\ \Pi_R(q_{M1}, q_{C1}) - \Pi_R^o$.

Proof of Proposition 4:
We show that $q_{C1}^I < q_{C1}^{\text{2PT}}$ for $\lambda > 0$.
First, we compare the equation systems

$$\frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} q_{C1} + P_M(q_{M1}, q_{C1}) - c = 0, \tag{15}$$
$$\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} q_{C1} + P_C(q_{M1}, q_{C1}) - c_1 - \delta\lambda \frac{\partial \pi_{I2}}{\partial c_2} = 0, \tag{16}$$

with the varied system

$$\frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} q_{C1} + P_M(q_{M1}, q_{C1}) - c + \delta\lambda \frac{\partial \pi_{M2}}{\partial c_2}|_{c_2^{\text{2PT}}} = 0, \tag{15'}$$
$$\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} q_{C1} + P_C(q_{M1}, q_{C1}) - c_1 - \delta\lambda \frac{\partial \pi_{I2}}{\partial c_2} + \delta\lambda \frac{\partial \pi_{M2}}{\partial c_2}|_{c_2^{\text{2PT}}} = 0. \tag{16'}$$

Here, the varied system represents the first-order conditions of the function $\Pi_I(q_{M1}, q_{C1}) + \delta\lambda \frac{\partial \pi_{M2}}{\partial c_2}|_{c_2^{\text{2PT}}} q_{M1} + \delta\lambda \frac{\partial \pi_{M2}}{\partial c_2}|_{c_2^{\text{2PT}}} q_{C1} + constant$, where *constant* as well as $\frac{\partial \pi_{M2}}{\partial c_2}|_{c_2^{\text{2PT}}}$ are constant, real values. Therefore, (15') and (16') characterize the maximum $(q_{M1}^{var}, q_{C1}^{var})$ of the new, varied objective function. As the latter two summands of the new objective function move the maximum outside (away from the origin), compared to the joint-profit maximum, the location of the maximum (of the varied function) is characterized by $q_{M1}^{var} > q_{M1}^I$ and $q_{C1}^{var} > q_{C1}^I$.

Second, we compare the varied system with

$$\frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} q_{C1} + P_M(q_{M1}, q_{C1}) - c - \delta\lambda \frac{\partial \pi_{M2}}{\partial c_2} (\frac{\partial q_{C1}/\partial w_{M1}}{\partial q_{M1}/\partial w_{M1}}) = 0, \tag{20}$$
$$\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} q_{C1} + P_C(q_{M1}, q_{C1}) - c_1 - \delta\lambda \frac{\partial \pi_{R2}^o}{\partial c_2} = 0. \tag{18}$$

This system characterizes the optimal quantity levels q_{M1}^{2PT} and q_{C1}^{2PT} in case of a two-part tariff, while the varied system characterizes the optimal levels q_{M1}^{var}, q_{C1}^{var} of the varied objective function. In case of the optimal quantity levels, solving the equation systems, the only difference (of the systems) is the multiplier $-\frac{\partial q_{C1}/\partial w_{M1}}{\partial q_{M1}/\partial w_{M1}} > 0$ in (20) with regard to the summand $\delta\lambda \frac{\partial \pi_{M2}}{\partial c_2}$. It is considered that the multiplier is smaller than 1 (as is the case whenever $|\frac{\partial^2 P_M}{\partial q_{C1} \partial q_{M1}} q_{M1} + \frac{\partial^2 P_C}{\partial q_{C1} \partial q_{M1}} q_{C1} + \frac{\partial P_C}{\partial q_{M1}} + \frac{\partial P_M}{\partial q_{C1}}| < |\frac{\partial^2 P_M}{\partial q_{C1}^2} q_{M1} + \frac{\partial^2 P_C}{\partial q_{C1}^2} q_{C1} + 2\frac{\partial P_C}{\partial q_{C1}} + \delta\lambda^2 \frac{\partial^2 \pi_{R2}^o}{\partial c_2^2}|$). Therefore, the optimal level q_{M1}^{2PT} is smaller than q_{M1}^{var}, and q_{C1}^{2PT} is larger than q_{C1}^{var}.

Altogether, $q_{C1}^{2PT} > q_{C1}^{var} > q_{C1}^{I}$. (As $q_{M1}^{2PT} < q_{M1}^{var}$ and $q_{M1}^{I} < q_{M1}^{var}$ there is no definite order for q_{M1}^{I} and q_{M1}^{2PT}.)

Proof of Lemma 5, and Proposition 6:

Solving by backwards induction, second-period profits are given by $\pi_{M2}(c_2)$ for M, and $\pi_{R2}^{o}(c_2)$ for R.

Downstream:

In the first period, the downstream firm R decides to accept M's contract, and discount condition, or rejects the offer, maximizing long-run profits

$$\Pi_R(q_{M1}, q_{C1}) = (P_M(q_{M1}, q_{C1}) - w_{M1})q_{M1} + (P_C(q_{M1}, q_{C1}) - c_1)q_{C1} + \delta\pi_{R2}^{o}(c_2)$$

where $c_2 = c_1 - \lambda q_{C1}$. As R is the only downstream firm, M will force R to accept the discount condition. Hence, the un-discounted wholesale price is as large as necessary to hinder R from accepting this un-discounted offer. When R rejects the offer, it earns Π_R^o. If R accepts the discount, its optimization problem is

$$max_{q_{M1}, q_{C1}} \Pi_R(q_{M1}, q_{C1})$$
$$s.t. \quad q_{M1} \geq q_{M1}^{*} \quad (Q)$$

where $w_{M1} = w_{Q1}$. In case of (Q) being non-binding, the first order conditions (17), (18) characterize the optimal quantity choice, leading to the same results as two-part tariffs. Therefore, we concentrate on the binding case, where $q_{M1} = q_{M1}^{*}$ and R maximizes only with respect to q_{C1}. This optimal downstream choice is now given by (18), where $q_{C1}(q_{M1}^{*})$ depends on M's first-period quantity, but does not depend on the wholesale price w_{Q1}. R accepts the discount when the following participation constraint is fulfilled:

$$(P_M(q_{M1}^{*}, q_{C1}(q_{M1}^{*})) - w_{Q1})q_{M1}^{*} + (P_C(q_{M1}^{*}, q_{C1}(q_{M1}^{*})) - c_1)q_{C1}(q_{M1}^{*}) - F_1^{Q} + \delta\pi_{R2}^{o}(c_2) \geq \Pi_R^{o}. \quad (21)$$

Upstream:

M will maximize profits Π_M with respect to q_{M1}^{*}, w_{Q1} and F_1^{Q}, subject to (21). Following the Kuhn-Tucker conditions, (21) is binding. Thus, the optimization problem can be simplified to

$$\max_{q_{M1}^{*}} \quad (P_M(q_{M1}^{*}, q_{C1}(q_{M1}^{*})) - c)q_{M1}^{*} + (P_C(q_{M1}^{*}, q_{C1}(q_{M1}^{*})) - c_1)q_{C1}(q_{M1}^{*}) + \delta\pi_2^{I}(c_2) - \Pi_R^{o}.$$

Note that the (simplified) optimization problem does not depend on w_{Q1}. The wholesale price as well as the fixed fee serve to reach R's acceptance. They can be substituted with respect to

$$F_1^{Q} = (P_M(q_{M1}^{*}, q_{C1}(q_{M1}^{*})) - w_{Q1})q_{M1}^{*} + (P_C(q_{M1}^{*}, q_{C1}(q_{M1}^{*})) - c_1)q_{C1}(q_{M1}^{*}) + \delta\pi_{R2}^{o}(c_2) - \Pi_R^{o}.$$

Differentiating the simplified optimization problem with respect to q_{M1}^{*}, we get

$$\frac{\partial P_M}{\partial q_{M1}^{*}} q_{M1}^{*} + \frac{\partial P_C}{\partial q_{M1}^{*}} q_{C1}(q_{M1}^{*}) + P_M(q_{M1}^{*}, q_{C1}(q_{M1}^{*})) - c - \delta\lambda \frac{\partial \pi_{M2}}{\partial c_2} \frac{\partial q_{C1}}{\partial q_{M1}^{*}}$$
$$+ \underbrace{\left(\frac{\partial P_M}{\partial q_{C1}} q_{M1}^{*} + \frac{\partial P_C}{\partial q_{C1}} q_{C1}(q_{M1}^{*}) + P_C(q_{M1}^{*}, q_{C1}(q_{M1}^{*})) - c_1 - \delta\lambda \frac{\partial \pi_{R2}^{o}}{\partial c_2} \right)}_{=0, \text{see (18)}} \frac{\partial q_{C1}}{\partial q_{M1}^{*}} = 0 \quad (22)$$

Note that $\frac{\partial q_{C1}}{\partial q^*_{M1}} = \frac{\partial q_{C1}/\partial w_{M1}}{\partial q_{M1}/\partial w_{M1}}$:
By using the implicit function theorem on (18), we get the derivation of q_{C1} with respect to q^*_{M1}:

$$\frac{\partial q_{C1}}{\partial q^*_{M1}} = -\frac{\frac{\partial^2 P_M}{\partial q_{C1}\partial q_{M1}} q^*_{M1} + \frac{\partial^2 P_C}{\partial q_{C1}\partial q_{M1}} q_{C1} + \frac{\partial P_C}{\partial q_{M1}} + \frac{\partial P_M}{\partial q_{C1}}}{\frac{\partial^2 P_M}{\partial q^2_{C1}} q_{M1} + \frac{\partial^2 P_C}{\partial q^2_{C1}} q_{C1} + 2\frac{\partial P_C}{\partial q_{C1}} + \delta\lambda^2 \frac{\partial^2 \pi^o_{R2}}{\partial c_2^2}}$$

The RHS of this equation equals the ratio between $\frac{\partial q_{C1}}{\partial w_{M1}}$ and $\frac{\partial q_{M1}}{\partial w_{M1}}$, also given by the implicit function theorem on (17), (18) (see proof of proposition 3).

That is, equation (22) is equal to (20), because $\frac{\partial q_{C1}}{\partial q^*_{M1}} = \frac{\partial q_{C1}/\partial w_{M1}}{\partial q_{M1}/\partial w_{M1}}$. Hence, the optimal quantity discount is characterized by (18), (20) as the two-part tariff, too. (Altogether, binding as well as non-binding quantity discounts lead to the same result as two-part tariffs.)[36] As a result, quantity levels equal the optimal levels in case of a two-part tariff. That is, learning-by-doing in case of quantity discounts is also larger than in case of maximum industry profits.

Proof of Lemma 7:
Solving by backwards induction, second-period profits are given by $\pi_{M2}(c_2)$ for M, and $\pi^o_{R2}(c_2)$ for R, again.

Downstream:
In the first period, the downstream firm has the choice to accept M's contract, and market-share discount condition, or reject the offer. Rejecting implies profits π^o_R for R. Accepting the contract is unprofitable in case of not-fulfilling the discount condition (for the same reasons as in case of quantity discounts). Fulfilling the discount condition, the downstream firm R maximizes profits $\Pi_R(q_{M1}, \frac{1-\rho_1}{\rho_1} q_{M1})$ with respect to q_{M1} and ρ_1 (where $\rho_1 = q_{M1}/(q_{M1} + q_{C1})$), and subject to $\rho_1 \geq \rho^*_1$. Note that the profit function is concave in q_{M1} and ρ_1, given by the concavity of $\Pi_R(q_{M1}, q_{C1})$ (in both quantities). As a non-binding discount condition leads to the same result as two-part tariffs, we are interested in the binding discount condition. The first-order condition, characterizing $q_{M1}(w_{\text{MS1}}, \rho^*_1)$, is

$$\begin{aligned}\frac{\partial \Pi_R}{\partial q_{M1}} &= \frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} \frac{1-\rho_1}{\rho_1} q_{M1} + P_M(q_{M1}, \frac{1-\rho_1}{\rho_1} q_{M1}) - w_{\text{MS1}} \\ &+ \frac{1-\rho_1}{\rho_1}\left(\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} \frac{1-\rho_1}{\rho_1} q_{M1} + P_C(q_{M1}, \frac{1-\rho_1}{\rho_1} q_{M1}) - c_1 - \delta\lambda \frac{\partial \pi^o_{R2}}{\partial c_2}\right) = 0. \quad (23)\end{aligned}$$

Upstream:
M maximizes profits with respect to w_{MS1}, ρ^*_1 and F_1 subject to the participation constraint, which is now given by

$$\begin{aligned}&(P_M\left(q_{M1}(w_{\text{MS1}}, \rho^*_1), \frac{1-\rho^*_1}{\rho^*_1} q_{M1}(w_{\text{MS1}}, \rho^*_1)\right) - w_{M1}) q_{M1}(w_{\text{MS1}}, \rho^*_1) \\ &+ (P_C\left(q_{M1}(w_{\text{MS1}}, \rho^*_1), \frac{1-\rho^*_1}{\rho^*_1} q_{M1}(w_{\text{MS1}}, \rho^*_1)\right) - c_1) \frac{1-\rho^*_1}{\rho^*_1} q_{M1}(w_{\text{MS1}}, \rho^*_1) + \delta\pi^o_{R2}(c_2) - F^{\text{MS}}_1 \geq \Pi^o_R\end{aligned}$$

[36]Note that the acceptance of the binding discount offer is given because the participation constraint is fulfilled. The downstream firm won't further deviate from q^*_{M1}, because it is already larger than R's preferable, optimal level of q_{M1}.

Here as well, the fixed fee serves to shift rents upwards. The participation constraint is hence binding. In contrast to the quantity discount however, the discounted wholesale price w_{MS1} as well as the share ρ_1^* will be used by the dominant firm to maximize R's profits. The simplified optimization problem can be written as

$$\begin{aligned}\max_{w_{\text{MS1}},\rho_1^*} & \left(P_M(q_{M1}(w_{\text{MS1}},\rho_1^*), \frac{1-\rho_1^*}{\rho_1^*} q_{M1}(w_{\text{MS1}},\rho_1^*)) - c\right) q_{M1}(w_{\text{MS1}},\rho_1^*) \\ & + \left(P_C(q_{M1}(w_{\text{MS1}},\rho_1^*), \frac{1-\rho_1^*}{\rho_1^*} q_{M1}(w_{\text{MS1}},\rho_1^*)) - c_1\right) \frac{1-\rho_1^*}{\rho_1^*} q_{M1}(w_{\text{MS1}},\rho_1^*) \\ & + \delta\pi_2^I(c_2) - \Pi_R^o\end{aligned}$$

The first order conditions are given by

$$\begin{aligned}\frac{\partial\cdot}{\partial w_{\text{MS1}}} =& \left\{ \frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} \frac{1-\rho_1^*}{\rho_1^*} q_{M1} + P_M(q_{M1}, \frac{1-\rho_1^*}{\rho_1^*} q_{M1}) - c \right. \\ & \left. + \frac{1-\rho_1^*}{\rho_1^*} \left(\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} \frac{1-\rho_1^*}{\rho_1^*} q_{M1} + P_C(q_{M1}, \frac{1-\rho_1^*}{\rho_1^*} q_{M1}) - c_1 - \delta\lambda \frac{\partial \pi_2^I}{\partial c_2} \right) \right\} \frac{\partial q_{M1}}{\partial w_{\text{MS1}}} = 0 \\ \frac{\partial\cdot}{\partial \rho_1^*} =& \left\{ \frac{\partial P_M}{\partial q_{M1}} q_{M1} + \frac{\partial P_C}{\partial q_{M1}} \frac{1-\rho_1^*}{\rho_1^*} q_{M1} + P_M(q_{M1}, \frac{1-\rho_1^*}{\rho_1^*} q_{M1}) - c \right. \\ & \left. + \frac{1-\rho_1^*}{\rho_1^*} \left(\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} \frac{1-\rho_1^*}{\rho_1^*} q_{M1} + P_C(q_{M1}, \frac{1-\rho_1^*}{\rho_1^*} q_{M1}) - c_1 - \delta\lambda \frac{\partial \pi_2^I}{\partial c_2} \right) \right\} \frac{\partial q_{M1}}{\partial \rho_1^*} \\ & - \frac{q_{M1}}{(\rho_1^*)^2} \left\{ \frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} \frac{1-\rho_1^*}{\rho_1^*} q_{M1} + P_C(q_{M1}, \frac{1-\rho_1^*}{\rho_1^*} q_{M1}) - c_1 - \delta\lambda \frac{\partial \pi_2^I}{\partial c_2} \right\} = 0\end{aligned}$$

where $q_{M1} = q_{M1}(w_{\text{MS1}},\rho_1^*)$.

Inserting (22) into the first-order conditions of M, we get

$$w_{\text{MS1}} - c - \delta\lambda \frac{1-\rho_1^*}{\rho_1^*} \frac{\partial \pi_{M2}}{\partial c_2} = 0 \tag{24}$$

$$\frac{\partial P_M}{\partial q_{C1}} q_{M1} + \frac{\partial P_C}{\partial q_{C1}} \frac{1-\rho_1^*}{\rho_1^*} q_{M1} + P_C(q_{M1}, \frac{1-\rho_1^*}{\rho_1^*} q_{M1}) - c_1 - \delta\lambda \frac{\partial \pi_2^I}{\partial c_2} = 0 \tag{25}$$

(13), (14), (23), (24) and (25) characterize the market-share discounts that M will offer. Moreover, inserting (24) and (25) into (23), yields (15). Furthermore (25) equals (16). Hence, (15) and (16) characterize the profit-maximizing outcome in case of the market-share discount. The optimal long-run profit of the dominant supplier is given by maximum industry profits minus R's outside option: $\Pi_I - \Pi_R^o$.

Following (24), the wholesale price is larger than marginal cost. In regard to (15) and (16), the optimal share ρ_1^* is equal to the joint-profit maximizing one. As ρ_1^* is larger than the share that R would prefer, the downstream firm would accept this market-share discount and would not have any ambitions to purchase a larger share of M's good. Altogether, the downstream firm will accept the market-share discount that yields maximum profits for M.

A.2 Robustness

A.2.1 M's learning-by-doing and network effects

Here, we suppose that the dominant upstream supplier's marginal cost decreases proportionally to the quantity sold in period 1:

$$c_{M2} = \max\ \{0, c - \lambda_M q_{M1}\}.$$

$\lambda_M > 0$ denotes M's learning parameter which does not need to equal the learning parameter of the competitive fringe. That is, we allow for different speeds of progress. The competitive fringe might for example face a larger learning parameter and might therefore present a potential threat for the dominant firm. To guarantee that the learning-by-doing progress is continuing, assume $\lambda_M \leq \frac{c}{q^I_{M1}}$, for q^I_{M1} being the quantity sold in case of maximum joint profits.

In this context, maximum industry profits depend on learning-by-doing of all upstream firms. Long-run joint profit is characterized by

$$\Pi_I = \pi_{I1}(q_{M1}, q_{C1}) + \delta\pi^I_2(c_{M2}, c_2)$$

where $\pi_{I1}(q_{M1}, q_{C1}) = (P_M(q_{M1}, q_{C1}) - c)q_{M1} + (P_C(q_{M1}, q_{C1}) - c_1)q_{C1}$ and second-period maximum industry profit depends on both c_2 and c_{M2}. Thus, the optimal quantities q^I_{M1}, q^I_{C1} in period 1 depend on the degrees of learning λ, λ_M.

In case of short-run contracts, we first observe the downstream firm's decision. In period 2, the downstream firm earns the outside option $\pi_{R2}(c_2)$, independent of conditional discounts. With respect to period 1, however the firm maximizes long-run profits $\Pi_R(q_{M1}, q_{C1})$ and accepts the dominant firm's contract offer only if profits at least equal the long-run outside option Π^o_R. As before, the single-period and long-run outside options depend on the fringe's good, respectively only on the fringe's learning effects. Maximizing profits, the downstream firm does not internalize the dominant firm's learning effects. Thus, by maximizing the dominant firm's profits, the same results occur as in section 4.1.

In case of long-run contracts, similar results occur as in section 4.2. When the downstream firm assumes that wholesale prices decrease with respect to first-period quantities, the joint-profit maximizing results are determined by the dominant firm setting the wholesale prices equal to marginal cost and earning profits via the fixed fee. Both learning-by-doing effects are internalized by the downstream firm and maximum profits are reached in all long-run contracts.[37] Indeed, these rebates lead to the joint-profit maximizing outcome in case of long-run contracts, where maximum joint profits are given by Π_I.

[37] This internalization occurs for example when the dominant upstream firm informs the downstream firm about its learning effects. A reasonable case for this information is granting a kind of growth-based rebates. To be precise, suppose the dominant firm could grant discounts that reduce the wholesale price with respect to the quantity already purchased (in previous periods). Note that these long-run results change when we differentiate between knowing about the decrease of costs and the decrease of wholesale prices. When the downstream firm cannot consider that M's wholesale prices decrease, the joint-profit maximizing outcome is not reached.