BGPE Intensive Course: Contracts and Asymmetric Information

Adverse Selection, Signaling, and Screening in Markets

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Market Failure

Stylized Facts:

- used cars, even if they are like new, sell far below their dealership price
- laid-off workers experience longer spells of unemployment than workers for different reasons without a job (e.g. military)
- private health care for the elderly is essentially unavailable
- young drivers pay very expensive insurance premiums

what do these empirical regularities have in common?

these markets are characterized by **informational asymmetries** and suffer from adverse selection

Productivity Uncertainty in the Labor Market

- 2 identical firms F, 1 representative worker W
- worker's productivity is $\theta \in [\underline{\theta}, \bar{\theta}]$, private information of W, c.d.f $F(\theta)$ is continuous and has full support
- utilities

$$u_W = wq + r(\theta)(1-q), \qquad u_F = (\theta - w)q$$

  where $q = 1$ if W works and $q = 0$ otherwise
  $w \in$ R = wage paid by firm
  $r(\theta)$ = W's reservation utility, $r' > 0$
- timing:
  - $t = 0$: W observes θ
  - $t = 1$: F offer fixed-wage employment contracts
  - $t = 2$: W accept/rejects $\rightarrow$ payoffs

Equilibrium Analysis

Equilibrium under Full Information

- both worker and firms know $\theta \rightarrow$ firms can offer productivity dependent wages $w(\theta)$
- competition between firms gives $w^*(\theta) = \theta$ in equilibrium
- W accepts employment if $w^*(\theta) = \theta \geq r(\theta)$
- equilibrium is efficient

Equilibrium under Asymmetric Information

- only W knows θ, F's only know $F(\theta)$
- assume $r(\underline{\theta}) \leq \theta$ and $r(\theta) < \theta, \ \ \forall \theta > \underline{\theta}$

$\Rightarrow$ efficient employment has $q(\theta) = 1 \ \forall \theta$

Equilibrium under Asymmetric Information

- firms do not know $\theta \Rightarrow$ offer (same) fixed wage contract w in equilibrium
- utility of worker of type θ who accepts wage offer of w is $u_W = w$
- comparing this with outside utility $r(\theta)$ gives supply

$$\Theta(w) = \{\theta |\ r(\theta) \leq w\}$$

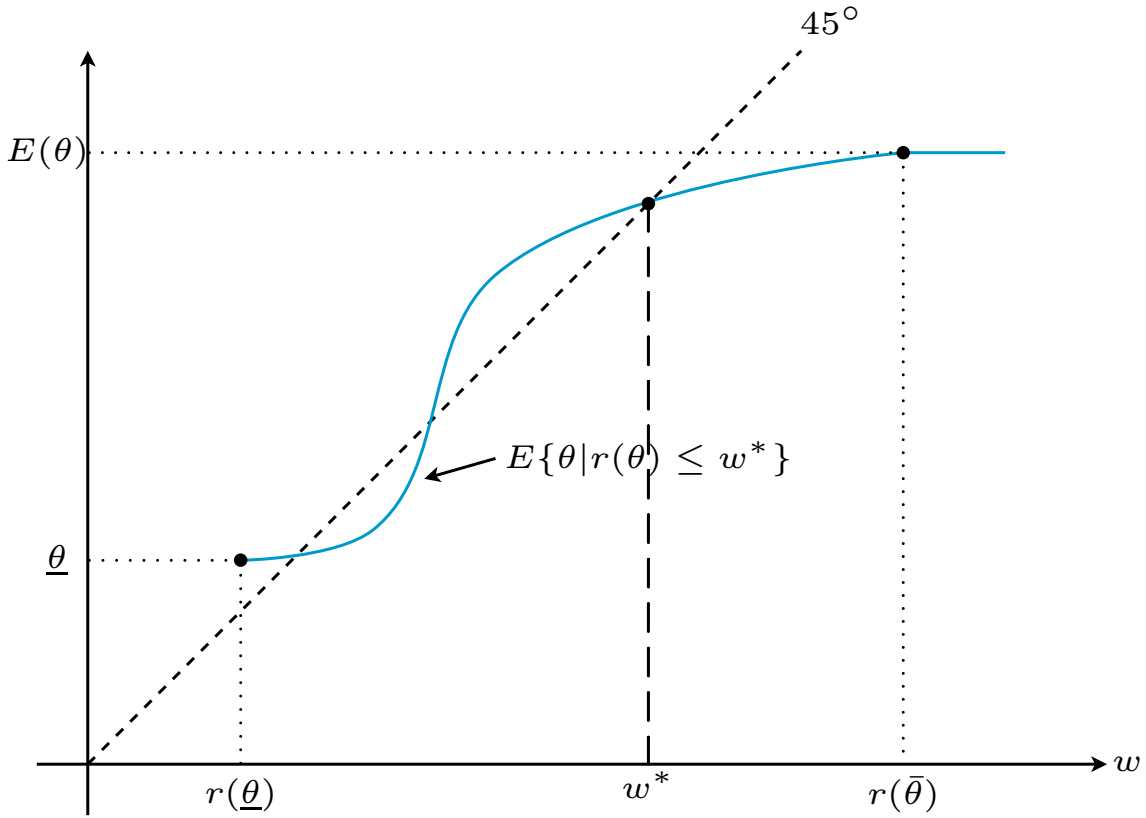
note: average productivity $E\{\theta|\theta \in \Theta(w)\}$ depends on price

- expected profit of firm in the market is $E\{\theta|\theta \in \Theta(w)\} - w$
- competition gives zero profits; necessary equilibrium condition is

$$w^* = E\{\theta|\theta \in \Theta(w^*)\} = E\{\theta|r(\theta) \leq w^*\}$$

Market Equilibrium

- graphic illustration:



Properties of the Equilibrium

- equilibrium exists
- equilibrium is generically unique: if there are multiple solutions to equation $w^* = E\{\theta|\theta \in \Theta(w^*)\} = E\{\theta|r(\theta) \leq w^*\}$, equilibrium is highest w satisfying condition
- welfare properties:
  - if $r(\overline{\theta}) \leq E(\theta)$, equilibrium is efficient
  - if $r(\overline{\theta}) > E(\theta)$, equilibrium is inefficient
  - if $r(\underline{\theta}) = \underline{\theta}$, can have complete market breakdown
- *intuition:* F cannot break even at wage $w = E(\theta) < r(\overline{\theta})$
  w falls $\rightarrow$ even more high-productivity workers drop out of market $\rightarrow$ wage must drop further
  (alternatively : market participation of individual worker introduces externality)

Government Intervention

Can free-market outcome be improved upon?

- assume: social planner/government does not know workers' productivity θ either (is subject to same informational constraints as firms)

$\rightarrow$ can only devise policies based on whether people choose to work or not (from IC's)

- let w_e (resp. w_u) be transfer to worker if $q = 1$ (resp. $q = 0$)

$$w_u + r(\hat{\theta}) = w_e \qquad (IC)$$

Claim. There is no (balanced) policy $\{w_e, w_u\}$ that yields to a Pareto improvement over the market outcome where workers with $\theta \leq \theta^*$ work and workers $\theta > \theta^*$ do not.

Government Intervention

Proof.

- equilibrium is Pareto efficient if $r(\overline{\theta}) \leq E(\theta)$
- if $r(\overline{\theta}) > E(\theta)$, let $\hat{\theta}$ be cut-off type worker given $\{w_e, w_u\}$
- budget balance requires

$$w_e F(\hat{\theta}) + w_u(1 - F(\hat{\theta})) = \int_{\underline{\theta}}^{\hat{\theta}} \theta f(\theta) d\theta = F(\hat{\theta}) E(\theta | \theta \leq \hat{\theta})$$

- substituting for $w_u + r(\hat{\theta}) = w_e$ gives

$$w_u(\hat{\theta}) = F(\hat{\theta})[E(\theta|\theta \leq \hat{\theta}) - r(\hat{\theta})]$$

$$w_e(\hat{\theta}) = F(\hat{\theta})[E(\theta|\theta \leq \hat{\theta}) - r(\hat{\theta})] + r(\hat{\theta})$$

- note: $\hat{\theta} = \theta^*$ gives market outcome $w_u = 0$ and $w_e = r(\theta^*)$
- $\hat{\theta} < \theta^*$ decreases social surplus
- suppose $\theta^* < \hat{\theta} < \bar{\theta}$. Since $w > w^* \Rightarrow E(\theta | r(\theta) \leq w) < w$ and $w^* = r(\theta^*)$ with $r' > 0$, we get $E(\theta | r(\theta) \leq r(\hat{\theta})) = E(\theta | \theta \leq \hat{\theta}) < r(\hat{\theta})$ and, hence, $w_u < 0$
  $\rightarrow$ unemployed are worse off
- $\hat{\theta} = \bar{\theta}$ implies $w_e = E(\theta) \rightarrow$ highest type workers are worse off since $r(\bar{\theta}) > E(\theta)$. $\square$

Other Markets with Adverse Selection

- formal model translates into goods markets and insurance markets:

labor		consumption good (cars)	insurance
workers	$\rightarrow$	sellers	insuree
firms	$\rightarrow$	buyers	insurer
θ	$\rightarrow$	buyer's value	-(exp. payments to insuree)
$r(\theta)$	$\rightarrow$	seller's value	-(inverse CE of risk)
w	$\rightarrow$	price	-(insurance premium)

- other markets where adverse selection relevant
  - credit market (θ = default risk of debtor)
  - dating and marriage market (θ = attractiveness of partner)
  - stock market and corporate equity market (IPO's) (θ = firm value)

Conclusion

adverse selection can lead to total market failure – if trade occurs, it will be less than efficient

- in markets with adverse selection (asymmetric information)
- prices are correlated with quality
- prices serve dual role of info transmission and market clearing
- insitutional/market responses against market failure caused by adverse selection
  - signaling and screening devices, e.g. warrantees
  - reputation (brand names and chains)
  - experts, inspections, standards, licensing
  - mandatory insurance (health, automobile)
  - liability laws

Using a Signal

asymmetric information causes market failure $\rightarrow$ participants have incentives to develop ways to reduce informational asymmetries

- signaling: informed market participants move first to **convey** info
- screening: uninformed market participants move first to **elicit** info

Signaling

- some market participants may be worse off as a result of their privately held information (sellers in lemons market, consumers in insurance market) $\rightarrow$ would want to reveal this information to others
- problem: information revealed must be credible $\rightarrow$ use of signals
- examples: warranties, lineups, peacock tail
- **but:** for the signal to work (be credible), it must be costly to fake

Education as a Signal in Labor Markets (Spence)

- 2 identical firms F, 1 representative worker W
- worker W with ability (=produtivity) $\theta \in \{\theta_L, \theta_H\}$, $\theta_L < \theta_H$, private information of W with $p = Prob\{\theta = \theta_H\}$
- set $r(\theta_H) = r(\theta_L) = 0$ for simplicity
- worker can invest in education $e \in \mathbf{R}_0^+$
  - marginal cost of education of θ_i-type is c_i with $0 < c_H < c_L$
  - education does **not** improve productivity θ
- utilities:

$$u_F = \theta_i - w \qquad \text{and} \quad u_W = w - c_i e \quad i = H, L$$

- timing:
  $t = 1$: W learns θ_i, chooses e
  $t = 2$: F's observe e, form beliefs $\mu(e) = Prop\{\theta = \theta_H | e\}$
  $t = 3$: F's offers (same) wage w

General Structure of Signaling (and Cheap Talk) Game

- dynamic game of incomplete information with sender S and receiver R
- timing

  $t = 0$ nature draws type $\theta_i \in \Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ for sender; with

  $$p_i = p(\theta_i) = Prob\{\theta = \theta_i\} > 0, \qquad \forall \theta_i \in \Theta$$

  $t = 1$ sender observes own type θ_i, chooses message

  $$m_j \in M = \{m_1, m_2, \ldots, m_N\}$$

  $t = 2$ receiver observes message m_j (but not θ_i) and chooses action $x \in X$

  $t = 3$ payoffs $u_S(x, m_j, \theta_i)$ and $u_R(x, m_j, \theta_i)$ realized

- signaling game: u_S depends on m_j
- cheap talk game: u_S (and u_R) is independent of m_j

Perfect Bayesian Nash Equilibrium

Definition A Perfect Bayesian Equilibrium (PBE) is a pair of strategies $m^*(\theta_i)$ and $x^*(m_j)$ and a belief $\mu^*(\theta_i|m_j)$ such that

a) $m^*(\theta_i)$ maximizes S's utility given R's strategy:

$$m^*(\theta_i) = \arg\max_{m_j \in M} u_S(x, m_j, \theta_i) \qquad \forall \theta_i \in \Theta$$

b) $x^*(m_j)$ maximizes R's utility given beliefs about S's type:

$$x^*(m_j) = \arg\max_{x \in X} \sum_{\theta_i} \mu(\theta_i|m_j) u_R(x, m_j, \theta_i),$$

c) R forms consistent beliefs that are calculated by Bayes Rule whenever possible:

$$\sum_{\theta_i \in \Theta} \mu(\theta_i|m_j) = 1, \forall m_j \qquad \mu(\theta_i|m_j) = \frac{p(\theta_i)}{\sum_{\theta_i \in \Theta(m_j)} p(\theta_i)}$$

where $\Theta(m_j) \equiv \{\theta_i \in \Theta | \, m^*(\theta_i) = m_j\}$.

Benchmark Cases

Equilibrium under Full Information

- **both** worker and firms observe ability θ_i, $i = H, L$
- firms offer wage $w_i = \theta_i$ independent of e

  $\rightarrow$ worker chooses $e_H = e_L = 0 \Rightarrow$ efficient

Imperfect but Symmetric Information

- **neither** worker **nor** firms observe θ_i, $i = H, L$

  $\rightarrow$ wage w can no longer depend on θ
- firms offer $w(e = 1) = w(e = 0) = p\theta_H + (1 - p)\theta_L$ independent of e

  $\rightarrow$ worker chooses $e_H = e_L = 0 \Rightarrow$ efficient

Imperfect and Asymmetric Information

- **only** worker **not** firms observe θ_i, $i = H, L$

  $\rightarrow$ wage w can no longer depend on θ

Separating Equilibrium

- two types of workers choose different education levels $e_H \neq e_L$

  $\rightarrow$ firms have beliefs $\mu(e_H) = \theta_H$ and $\mu(e_L) = \theta_L$

  $\rightarrow$ firms offer wages $w(e_H) = \theta_H$ and $w(e_L) = \theta_L$

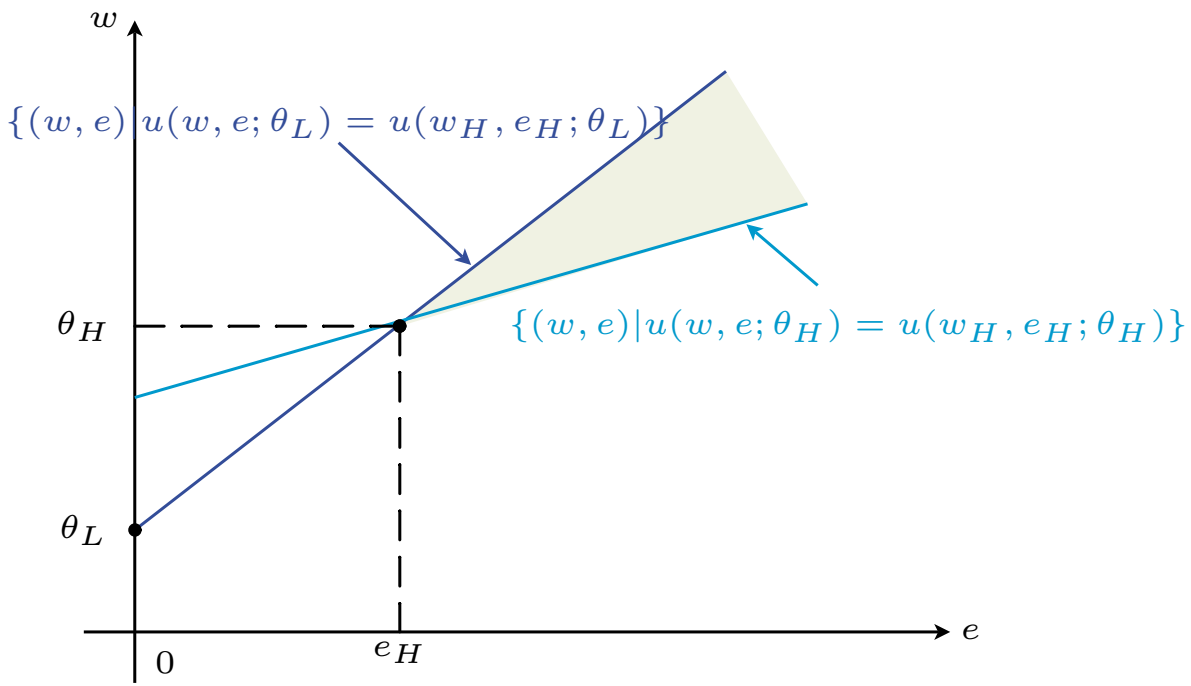
- $e_H \neq e_L$ optimal for W requires (from (IC)'s) $e_L = 0$ and $e_H > 0$ where

$$\boxed{c_H e_H < \theta_H - \theta_L \leq c_L e_H} \qquad (\star)$$

- off equilibrium beliefs? ... let $\mu(e) = 0$ for $e < e_H$, $\mu(e) = 1$ for $e \geq e_H$

Education as a Signal

- graphic illustration of a separating equilibrium:



- note: W's preferences satisfy single crossing property:

$$\frac{dw}{de}|_{\bar{u}} \quad \uparrow \text{ in } \quad \theta \quad \text{or} \quad u_e(w,e;\theta_L) - u_e(w,e,;\theta_H) = c_L - c_H > 0 \quad \text{(SC)}$$

Other Types of Equilibria

Pooling Equilibrium

- both types of workers choose same education level $e_H = e_L = \tilde{e}$

  $\rightarrow$ firms have beliefs $\mu(\tilde{e}) = p\theta_H + (1-p)\theta_L$

  $\rightarrow$ firms offer wage $w(\tilde{e}) = \theta_H + (1-p)\theta_L$
- off equilibrium beliefs? ... let $\mu(e) = 0$ for $e < \tilde{e}$, $\mu(e) = p$ for $e \geq \tilde{e}$
- note: both $\tilde{e} = 0$ and $\tilde{e} > 0$ may be possible

Semi-Separating Equilibrium

- some education levels are chosen by both types, and some by only one type
- no type chooses more than two levels of e with positive probability
- three types of semi-separating equilibria, depending on who makes two different choices with positive probability

Properties of Signaling in Markets

- equilibrium with $e_H = e_L = 0$ always exists
- wasteful signaling in separating equilibria where $e_H > 0$
- wasteful education also possible in some pooling equilibria where $e_H = e_L > 0$
- multiple (types) of equilibria cannot be unambiguously ranked by the Pareto criterion
- out-of-equilibrium beliefs $\mu(e)$ are critical but not always plausible $\rightarrow$ **equilibrium refinements**

Intuitive Criterion (Cho-Kreps): if an out-of-equilibrium deviation is dominated for some type θ_i, but not for all types, then out-of-equiloibrium beliefs should put zero probability of that type

action "dominated" = **whatever** the receiver's action to the deviation, type θ_i is made worse off by the deviation

$\rightarrow$ selects **unique** least-cost separating equilibrium

this equilibrium may be Pareto dominated, however (intuition = informational externality)

Conclusion

signaling can least to wasteful resource allocation and the market outcome may thus be inefficient

- in markets with signaling
  - privately informed individuals use signal to reveal their information
  - signal only works (is credible) if sending the same signal is too costly for other individuals
- other markets where wasteful signaling is relevant
  - consumer products (signal = warranty, advertisements, price)
  - corporate equity and start-ups (signal = equity/own money invested)
  - legal disputes (signal = pre-trial settlement demands)
  - bargaining (signal = rejection of offer/delay)
  - live entertainment and restaurants (signal = lineups)
  - marriage and dating (signal = fancy car)
  - poker (signal = stakes)

Competitive Screening

Screening = contractual arrangements originating from uniformed side of market to elicit information from informed market participants

- 2 identical firms F, 1 representative worker W
- worker W with ability (=produtivity) $\theta \in \{\theta_L, \theta_H\}$, $\theta_L < \theta_H$, private information of W with $p = Prob\{\theta = \theta_H\}$, $r(\theta_i) = 0$
- firm can set task difficulty level $t \in \mathsf{R}_0^+$
- utilities: $u_F = \theta_i - w$ and $u_W = w - c_i t \quad i = H, L$
- task difficulty does not influence productivity and costs workers effort c_i with $0 < c_H < c_L$
- timing:
  $t = 1$: W learns θ_i
  $t = 2$: F's offer menu of contracts $\{(w, t)\}$
  $t = 3$: W picks firm/contract

Equilibrium Analysis

- note: W's preferences satisfy single crossing property:

$$\frac{dw}{dt}|_{\bar{u}} \quad \uparrow \text{ in } \quad \theta \quad \text{or} \quad u_t(w,t;\theta_L) - u_t(w,t,;\theta_H) = c_L - c_H > 0 \quad \text{(SC)}$$

Equilibrium under Full Information

- **both** worker and firms observe ability θ_i, $i = H, L$
- firms offer wage/task contracts $(w_i, t_i) = (\theta_i, 0)$

  $\rightarrow$ worker accepts employment $\Rightarrow$ SPE is efficient

Imperfect but Symmetric Information

- **neither** worker **nor** firms observe θ_i, $i = H, L$

  $\rightarrow$ wage w can no longer depend on θ
- firms offer wage/task contract $(w, t) = (p\theta_H + (1-p)\theta_L, 0)$

  $\rightarrow$ worker accepts $\Rightarrow$ efficient

Imperfect and Asymmetric Information

- **only** worker **not** firms observe θ_i, $i = H, L$

  $\rightarrow$ wage w can no longer depend on θ

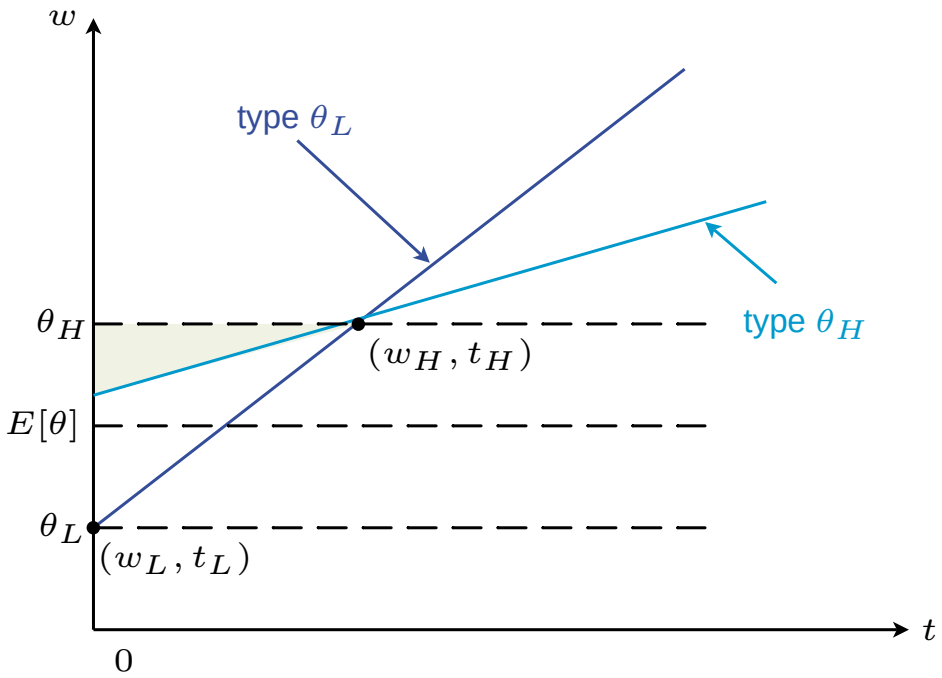
find equilibrium in steps

- let (w_H, t_H) and (w_L, t_L) be the lowest-wage contracts accepted by workers of type i in equilibrium
- Step 1: in any SPE, firms earn zero profits
- Step 2: there is no SPE in which the high-type worker accepts a wage $w_H < \theta_H$ with positive probability

  $\rightarrow$ all low types must earn $w_L = \theta_L$ and the equilibrium must be separating
- in a separating SPE, must have $t_L = 0$ and $\theta_H - c_L t_H = \theta_L$, i.e., the only candidate for equilibrium is the least-cost separating one

Screening with Task Assignments

- graphic illustration of the separating equilibrium:



- note: no deviation profitable; in particular, any contract attracting both types of workers lies above break even line

Screening with Task Assignments

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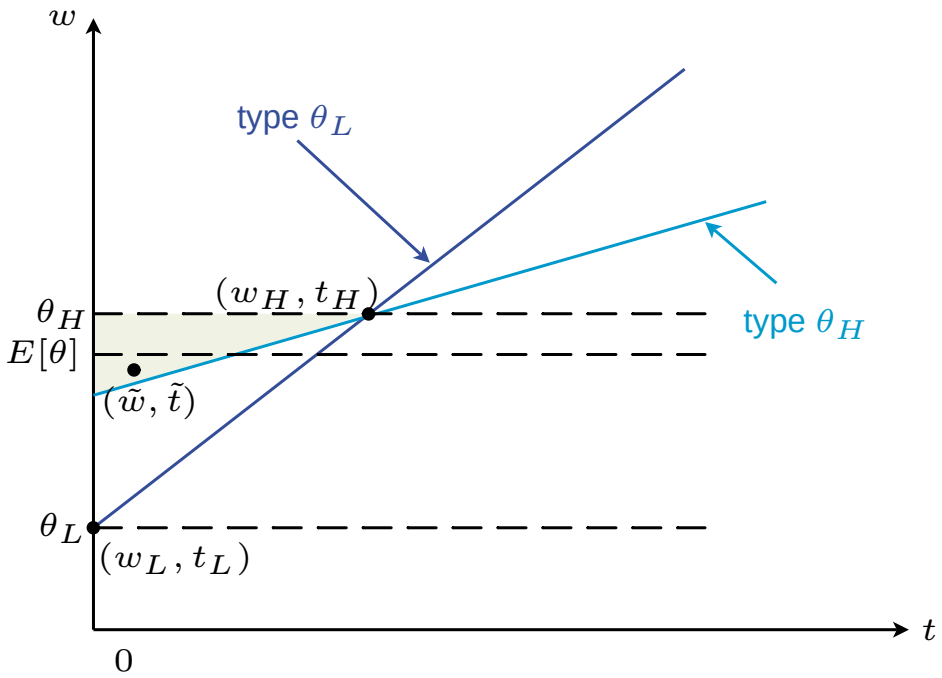

- note: deviation profitable; a contract $(\tilde{w}, \tilde{t})$ attracting both types of workers lies below break even line
- if least-cost separating equilibrium is Pareto dominated by pooling, then no equilibrium exists

Properties of Market Screening

- equilibrium (in pure strategies) may not exist
- if equilibrium exist, is unique and identical to least-cost separating equilibrium
- equilibrium separation is necessary; otherwise, rival firms can do "cream skimming" and attract only high-types away form the firm

$\rightarrow$ wasteful screening with $t_H > 0$ occurs in equilibrium $\rightarrow$ outcome is Pareto inefficient

- other screening devices in labor markets
  - probation periods
  - seniority wages
  - performance based compensation

Conclusion

sceening can least to wasteful resource allocation and the market outcome may thus be inefficient

- in markets with screening
  - uninformed individuals use screening devices to make informed individuals reveal their information by choice of (preferred) contract from menu
  - screening device only works (separates types) if accepting same contract is undesirable for other individuals
- other markets where wasteful screening is relevant
  - consumer products (screening device = warranty, price)
  - credit (screening device = collateral, incomes)
  - marriage and dating (screening device = household chores)
  - poker (screening device = stakes)