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Project risk neutrality in the context of asymmetric information

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Abstract

Using the modelling framework of Stiglitz & Weiss (1981), we show that – perhaps surprisingly – there is no influence of projects' riskiness on the capital market equilibrium. The savings interest rate fully determines the amount of credit rationing and the nature of an equilibrium (adverse selection, two-prices etc.). This rate is, in turn, fully determined by the relative probabilities of success of firms' projects (and, thus, repayment of their debt). Hence, making capital markets overall "less risky", which may for example be the case when financial markets become greener, does not alleviate concerns of asymmetric information. The result holds both for cases of hidden information and for those of hidden actions.

JEL classification: D82, G14, G21

Key words: Asymmetric Information, Financial Markets, Green Loans, Hidden Information, Hidden Action, Project Risk

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1 Introduction

Recent literature suggests that banks financing green firms and green projects, respectively, face a lower risk of default (see, for example, Cui et al., 2018 or An & Pivo, 2020). At the same time, evaluating the performance of fixed-interest green financial products mostly reveals a (positive) "greenium", i.e. lower returns compared to similar non-green (henceforth, brown) counterparts (Cheong & Choi, 2020). At first sight, these observations combined seem to prompt that, once the transition from a brown to a green economy is completed, the way for a debt market with few to no problems of intermediation resulting in credit rationing and other inefficiencies is paved. We discourage this hypothesis by inspection of the famous Stiglitz-Weiss (SW) model (Stiglitz & Weiss, 1981).

Despite the fact that much work has focused on, expanded, and sometimes even corrected said model, there is only little discussion of the role of overall project riskiness yet. However, its inspection yields quite a grand result which roughly goes like the following. Start from any capital market equilibrium in the SW model. Now apply an equi-proportionate mean-preserving change to the success probabilities of all projects availabe for conduction by firms. Then the result is always an analagous equilibrium with identical savings interest rates and the same amount of capital allotted to the same types of firms. Thus, the absolute level of project riskiness is neutral.

To the best of our knowledge, the only similar contribution on the role of probabilities in this model appears to stem from Stiglitz & Weiss (1992) themselves. Their analysis on success probabilities in a very related model setup, however, produces results that are qualitatively different from ours due to a difference in assumptions. We shall address this matter in more detail later and aim to induce an open discussion on the subject.

Before proceeding to our hypothesis, noteworthy contributions on the Stiglitz-Weiss model are reviewed in Section 2 and the model is briefly set up in Section 3. We begin the actual model analysis in Section 4 by analyzing the benchmark case without asymmetric information. The latter is introduced in Section 5 by assuming different project endowments of firms which are not distinguishable to outsiders. This corresponds to the case of hidden information. Hidden action with non-enforceable project choice of corporates, on the other hand, is analyzed in Section 6. Each of the three last-mentioned Sections is accompanied by a numerical example. We briefly discuss neutrality of the method of financing by inspecting trade in shares rather than fixed-interest loans in Section 7. Section 8 concludes the paper.

2 Literature Overview

As a look into the References Section reveals, much related work originates from Stiglitz and Weiss themselves. A somewhat definitive paper among these is Stiglitz & Weiss (1992). There, the model is set up in a slightly different, yet largely parallel way. It aims to consider the credit market under the lense of a macroeconomic model where the project owners are specified in more detail. They have rich or poor endowments, decreasing absolute risk aversion and the choice between safe and risky projects. As a result, both adverse selection and moral hazard are integrated into the model: The rich switch to the riskier project already at lower interest rates than the poor while the latter may be rationed more in equilibrium. Banks offer contracts as combinations of interest rate and collateral. Various types of equilibria can arise (pure rationing, two-prices etc.).

Suarez & Sussman (1996) integrate the SW-model into a dynamic setting, namely one of overlapping generations (OLG). There, firms produce output deterministically in one period, but stochastically in the subsequent one. Consequently, an alternating price structure – which can be an endogenous result of that model – of low and high prices (induced by booms and busts, respectively) leads to intergenerational disparities. As is standard in OLG-models, a redistribution scheme can be found that enhances welfare.

A contribution by Coco (1999) employs risk aversion of firms in the model as well. Accordingly, project owners are considered as "entrepreneurs" (individuals with risk preferences) rather than faceless conglomerates there. He identifies risk aversion itself as a possible driver of credit rationing as the risk averse owners of relatively safe projects are the first victims of adverse selection. Their reluctance to post collateral (which is also endogenous in Coco's (1999) version of the model) exacerbates this problem.

Coco (1997) and Arnold & Riley (2009) show independently of each other that the return on lending achieves its unique global maximum at the highest possible credit interest rate which still produces positive capital demand. At that rate, the riskiest firms are just indifferent between applying for a loan to conduct their project and staying inactive. Thus, all the (expected) project return goes to lenders which can then, in turn, pay savers the entire (expected) project return as the savings interest rate. Since all projects are expectationally equivalent, this is impossible with lower credit rates at which the riskiest corporates make positive profits.

Arnold et al. (2014) analyze non-diversifiable risk and risk aversion of savers. Employing the former, individuals cannot be guaranteed a definite rate of return on their deposits. Rather, different states of nature have to be distinguished. Consumers dislike the resulting uncertainty and, thus, tend to save less, which gives room to additional credit rationing in equilibrium.

Su & Zhang (2017) again endogenize collateral in addition to interest rates of a contract, just as Stiglitz & Weiss (1992) and Coco (1999). In their work, just like in the first-mentioned, it serves the purpose of allowing for the co-existence of both adverse selection and moral hazard. They find that the two phenomena can co-exist when there is second-order stochastic dominance of some projects against others. The intuition is that for a given (relatively high) rate of repayment, potential borrowers that can choose from a "good" (not too risky) pool of projects may not demand capital at all while those with the ability to choose from some other "bad" (more risky) pool will pick a very risky alternative among these (to ensure some positive net profit in case of a success, say).

3 Model

The notation mostly corresponds to the one used in Arnold et al. (2014), stripped of the time dimension. It is also the one used in Chapter VII of Arnold's (2020) textbook, which the following exposition follows closely.

On the capital demand side, we have N_j firms of each risk class j. More precisely, each risk class encompasses a continuum of length N_j of corporates. This continuum assumption ensures that capital lenders can take expected values as definitive payoff due to perfect diversification (given uncorrelated failure events) as the strong law of large numbers applies. Hence, financial intermediaries do not bear any risk, so we do not have to specify their risk preferences.

Each project is all-or-nothing in the sense that it has a probability of success p_j, in which case it delivers a return R_j, while with the complementary probability $1-p_j$, its payoff is zero. The risk classes are ordered by decreasing success probabilities, i.e. $0 < p_J < p_{J-1} < ... < p_1 < 1$. However, no project is unambiguously superior to another as they are all expectationally equivalent: $p_j R_j = E(R), j = 1, ..., J$. In other words, projects are mean-preserving spreads from each other. It follows that $R_1 < R_2 < ... < R_J$. In order to finance their corresponding project, firms need up-front investment capital B the only potential source of which is a loan from a bank. Projects are assumed to be socially desirable in the sense that $E(R) > B$.[1] In exchange for the borrowed amount, firms pledge collateral C to the bank and promise to repay their debt including interest rate r. We adopt the assumption of exogenous collateral in order to simplify the analysis. Wette (1983) shows that, even with risk neutral rather than risk averse borrowers, adverse se-

[1] Note that this additionally implies $R_1 > B$ which is equivalent to some capital demand by every firm in the plausible interval of credit rates $r \geq 0$.

lection problems are exacerbated via additional pledged collateral.[2] That state of affairs serves to diminish expected lender profits. As this leaves credit interest rates as the more attractive rationing device, modelling the former as the single latter suffices. If corporates fail to repay, the collateral is withheld by the bank. Of course, firms could also choose to self-finance in order to avoid paying interest. This case is, however, not of huge interest as it would limit focus on large firms (those capable of self-financing) and essentially eliminate the role of risk in the model. To close this channel, we assume $C < B$. To sum up, firm profits follow

$$E(\pi_j^F) = p_j[R_j - (1+r)B] + (1-p_j)(-C). \tag{1}$$

On the financial intermediary side, there are banks charging a fixed rate of interest r on the loans they give out. To begin with, assume that these credit interest rates can be tailored to each risk class of firms at first (in Section 4). Afterwards, we make financial intermediaries completely unable to observe project riskiness. Hence, the interest charged on loans can no longer be firm-specific. This is where asymmetric information finds its way into the model. If debt cannot be served, lenders[3] can claim the pledged collateral. Thus, their expected repayment payoff from financing a type-j firm takes the form

$$E(\pi_j^L) = p_j(1+r)B + (1-p_j)C. \tag{2}$$

Finally, there is also a capital supply side of the model. Individuals provide savings to the financial intermediaries in exchange for some savings interest rate i. Their capital supply function can take any arbitrary shape as long as it is increasing in i:

$$S(i), \ \frac{dS(i)}{di} > 0 \tag{3}$$

[2]Stiglitz and Weiss themselves acknowledge this contribution to their model (cf. Stiglitz & Weiss, 1983, p. 914 and Stiglitz & Weiss, 1992, p. 694).

[3]We use the superscript L for "lenders" to refer to banks in order to avoid confusion as B could mean "borrower" – a firm – as well. Additionally, the symbol is already in use for the borrowed amount of capital.

Banks cannot choose this rate haphazardly. Rather, they are driven down to zero profits by Bertrand competition (cf. Coco, 1999, p. 562), which is the standard result of price competition. In a more general setting of repeated capital allocation, Stiglitz & Weiss (1983) argue that banks indeed cannot make positive profits when forced to attract both borrowers and depositors (cf. pp. 919-920). Arnold (2012) shows that this is the only Nash equilibrium of mutually interdependent bank behavior (cf. p. 222). Our analysis of variations in success probabilities will take a specific form: They are always assumed to change by $100x\%$. More precisely, we resort to the special case of

$$p'_j = (1+x)p_j, \ j = 1, ..., J.$$

While this may look like a stark restriction at first, it clearly disentangles the hypothesis intended to be proven from another effect, namely risk dispersion. We inspect only proportionate changes in all risk classes because it is indeed our goal to analyze effects of the overall level of project riskiness. Further, note that expected revenues remain fixed at $E(R)$:

$$R'_j = \frac{1}{1+x} R_j, \ j = 1, ..., J.$$

4 Complete Markets

The analysis of no information asymmetries at all is meant to serve as a benchmark and justify the further analysis. The equilibrium with complete markets turns out to be fairly simple.

4.1 Equilibrium with Complete Markets

As lenders have to make zero profits overall and borrowers are distinguishable, lender profits have to equal interest services to savers for any project financed. They choose r firm-specifically such that, with each class j, they have expected (and, by continuity, definitive) profits of

$$E(\pi_j^L) = p_j(1+r)B + (1-p_j)C = (1+i)B. \tag{4}$$

Furthermore, it is straightforward that the profits of borrowers and lenders have to add up to the total project return[4], i.e.

$$E(\pi_j^F) + E(\pi_j^L) = E(R). \tag{5}$$

Together, (4) and (5) imply

$$E(\pi_j^F) = E(R) - (1+i)B. \tag{6}$$

Hence, neither firm profits nor bank profits depend directly on r. The latter is set in such a way that the differences in p_j among risk classes are completely irrelevant. More precisely, manipulation of (4) yields

$$r = \frac{1+i}{p_j} - \frac{1-p_j}{p_j}\frac{C}{B} - 1 \tag{7}$$

which is uniquely determined by i for each firm. Furthermore, it reveals that firms conducting safer projects pay lower interest rates as the derivative

$$\frac{\partial r}{\partial p_j} = -\frac{1+i-C/B+p_j^2}{p_j^2}$$

is negative due to $C < B$.

Determining the capital market equilibrium requires equating capital supply and demand. Capital demand is simply the required capital of all firms taken together as long as they are not drained the entire expected project return:

$$I(i) = \begin{cases} \sum_{j=1}^{J} N_j B, & \text{for } i \leq \frac{E(R)}{B} - 1 \\ 0 & \text{else} \end{cases}. \tag{8}$$

Equating this with $S(i)$ yields the equilibrium savings rate i and, thus, (implicitly) a value of r for each firm.

[4]One can also obtain this result from adding up (1) and (2).

To return to the hypothesis of this paper, what is the effect of changes in p_j on the capital market equilibrium? For $E(R)$ constant, which we will assume throughout, the answer is plainly: nothing. i is determined independently of r at first. Here is where the type of equilibrium is fixed: Either there is full allotment of capital or, if savers demand to high saving rates, the entire project return goes to lenders (who pass it through to savers) and credit is rationed by $I((E(R)/B)-1) - S((E(R)/B)-1)$. As this expression only depends on the p_j's via $E(R)$, it is constant for mean-preserving changes in project riskiness. In fact, changes in p_j may so far even be arbitrary. This goes to show that there is room for a neutrality result in versions of the model with asymmetric information as well.

4.2 Complete Markets Equilibrium: Example

The numerical example in this and the following sections will restrict itself to two risk classes. While this may seem like an oversimplification at first, Stiglitz & Weiss (1987) argue that the number of borrower types is often irrelevant for the amount of rationing in equilibrium. We thus content ourselves with the minimum number of types necessary to obtain equilibria with adverse selection or moral hazard, namely two, for expositional ease.

Let the two firm or project types be characterized by $p_1 = 0.8$, $R_1 = 125$, $p_2 = 0.5$ and $R_2 = 200$. There are $N_1 = N_2 = 250$ firms of each type. Hence, the expected payoff is $E(R) = 100$. Firms need to finance a capital input of $B = 80$ for which they can pledge collateral of $C = 50$. Capital demand is $S(i) = 200\,000i$.

The entire project return of $(E(R)/B)-1$ here takes on the value 0.25. By (8), we have capital demand of $(250 + 250) \times 80 = 40\,000$ up to that value. Equilibrating demand and supply yields an interior equilibrium where $i^* = 0.2$. By (7) we know that type-1 firms pay an interest of

$$r = \frac{1+0.2}{0.8} - \frac{1-0.8}{0.8} \times \frac{50}{80} - 1 = \frac{11}{32}$$

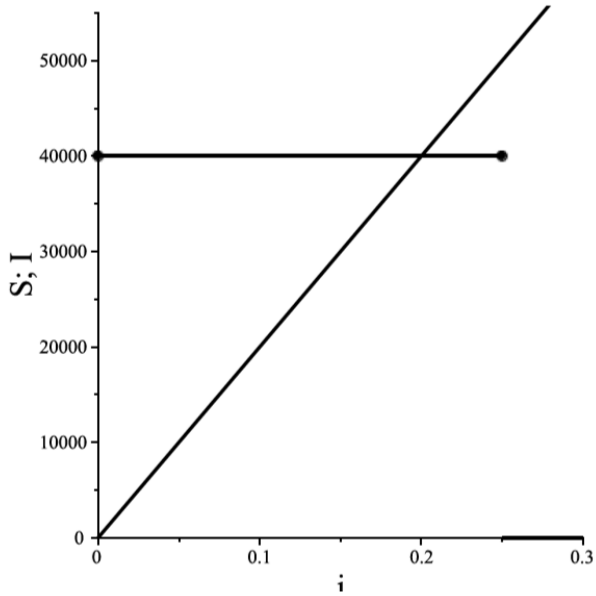

Figure 1: Equilibrium with complete markets

on their loans, while type 2 has to provide more:

$$r = \frac{1+0.2}{0.5} - \frac{1-0.5}{0.5} \times \frac{50}{80} - 1 = \frac{31}{40}.$$

A visualisation of this equilibrium is viewable in Figure 1.

Turning to changes in the p_j's, we set $x = 0.1$ such that $p'_1 = 0.88$, $R'_1 = 1\,250/11$, $p'_2 = 0.55$ and $R'_2 = 2\,000/11$. Note that $E(R') = 100$ still.

All of the steps up to and including the equilibration of demand and supply follow the deduction above verbatim. Thus, Figure 1 posits a valid representation of this altered version of the economy as well. We only obtain different implied interest rates on loans, namely

$$r' = \frac{1+0.2}{0.88} - \frac{1-0.88}{0.88} \times \frac{50}{80} - 1 = \frac{49}{176}$$

for firms of the first type and

$$r' = \frac{1+0.2}{0.55} - \frac{1-0.55}{0.55} \times \frac{50}{80} - 1 = \frac{59}{88}$$

for those of the second type.

5 Hidden Information

In this Section, borrowers are at an informational advantage: they know something the lenders don't, namely their own risk class. The odd result that this can be disadavantageous for some of the borrowers is at the core of the original Stiglitz & Weiss (1981) paper. The general idea dates back at least to Akerlof (1970).

5.1 Equilibrium with Adverse Selection

We proceed with a version of the model where there are still J types of borrowers. Those are, however, not distinguishable from the lenders' point of view. This means that there can only be one single interest rate r charged by banks on all firms' loans. Hence, while firm profits are still given by (1), equation (2) now only gives lender profits from one single debt contract with a type-j borrower. Therefore, p_j has to be replaced by an average success probability. This depends, of course, on the mix of borrowers applying for a loan. We can derive the condition for every firm type j to demand capital from

$$E(\pi_j^F) \geq 0$$

as 0 is the outside option of each firm. With (1), we obtain

$$r \leq \frac{E(R) - (1 - p_j)C}{p_j B} - 1 \equiv r_j. \tag{9}$$

This threshold clearly depends negatively on p_j:[5]

$$\frac{\partial r_j}{\partial p_j} = -\frac{1}{B p_j^2}(E(R) - S) < 0. \tag{10}$$

Hence, it is the owners of safe projects that can tolerate only lower interest rates before leaving the capital market. This is precisely the core of the adverse selection problem. It reveals that expected success probabilities $E(p_j | r \leq r_j)$ experience

[5] Remember $E(R) > B > C$ and $p_j \in (0; 1)$, $j = 1, ..., J$.

non-monotonous jumps downwards at each r_j. Therefore, lender profits

$$E(\pi^L|r \leq r_j) = E(p_j|r \leq r_j)(1+r)B + (1 - E(p_j|r \leq r_j))C \tag{11}$$

are characterized by upward sloping segments in their dependence on r with discontinuous negative jumps at the critical r_j's. As both Coco (1997) and Arnold & Riley (2009) show, this function reaches its global maximum at r_J. Thus, so does the percentage return

$$i(r) = \frac{E(\pi^L|r \leq r_j)}{B} - 1$$

which is paid to savers as deposit interest rate. Its maximum value is, of course, total project return[6]

$$i(r_J) = \frac{E(R)}{B} - 1.$$

What follows now is the actual analysis of the influence of project risk on profits for both sides of the market and, thus, the capital market equilibrium. Along the same line of thought that determined $i(r_J)$, we know that each project j yields zero profit to its owner at the corresponding r_j: Denoting lender profits from financing this project as in (2), the former are thus given by $E(\pi_j^L|r = r_j) = E(R)$. Risk classes $1, ..., j-1$ at this point have already left the market. Riskier projects' owners $j' \in \{j+1, ..., J\}$ generate expected profits

$$E(\pi_{j'}^F|r = r_j) = p_{j'}(R_{j'} - (1+r_j)B) + (1 - p_{j'})(-C).$$

Plugging in (9), we obtain

$$E(\pi_{j'}^F|r = r_j) = (1 - \frac{p_{j'}}{p_j})(E(R) - C). \tag{12}$$

We arrive at the conclusion that borrower profits at critical interest rates do not depend on their individual success probability per se, but only on the latter in relation to that of the last just-not-rationed project. Hence, varying the success probabilities p_j of all firms proportionately is sufficient to obtain identical profits

[6]To see this, either plug r_J into (11) or simply remember $E(R) = E(\pi_j^L) + E(\pi_j^F)$.

for them. By (5), the same automatically also holds for bank profits from financing that very firm. They are given as

$$E(\pi^L_{j'}|r = r_j) = \frac{p_{j'}}{p_j}E(R) + (1 - \frac{p_{j'}}{p_j})C.$$

The percentage return on the loan to a firm of type j' is then

$$i_{j'} = \frac{E(\pi^L_{j'}|r = r_j)}{B} - 1 \tag{13}$$

and, trivially, $i_j = E(R)/B - 1$. Now $i(r_j)$ can be computed simply as an average value:

$$i(r_j) = \frac{\sum_{k=j}^{J} N_k i_k}{\sum_{k=j}^{J} N_k}. \tag{14}$$

If we switch to the p'_j-regime, the critical interest rates are now given by

$$r'_j = \frac{E(R) - (1 - (1 + x)p_j)C}{(1 + x)p_j B} - 1.$$

They clearly differ from (9) due to the changes in the p_j which are unambiguously inversely related to r_j, see (10). At this new critical intrest rate, firm j obviously makes zero profits again. For all other corporates (those in j'), we get

$$E(\pi^{F'}_{j'}|r = r'_j) = (1 + x)p_{j'}(R'_{j'} - (1 + r'_j)B) + (1 - (1 + x)p_{j'})(-C)$$

which collapses to the same expression as (12). By the same argument as before, we obtain bank profits identical to their previous level, $E(\pi^{L'}_{j'}|r = r'_j) = E(\pi^L_{j'}|r = r_j)$ and, hence, the same relative return $i_{j'}$. It follows that, albeit the credit interest rates r_j where the jumps happen change, the pooled return $i(r_j)$ available to banks as a savings interest rate they can offer individuals remains the same at all those critical credit interest rates. This inisght proves to be an important corollary in our venture to show that the wealth distribution between borrowers and lenders along with the amount of rationing is unaltered not only at those critical rates, but at any equilibrium.

Before analyzing the equilibrium, we first have to close the model by specifying its demand side. Firms will obviously wish to conduct their projects as long as those are lucrative in expectations. Non-negative firm profits $E(\pi_j^F) \geq 0$ range until critical interest rates r_j. Hence, capital demand is given by

$$I(r) = \begin{cases} \sum_{k=1}^{J} N_k B, & \text{for } r \leq r_1 \\ \sum_{k=j}^{J} N_k B, & \text{for } r_{j-1} < \ r \leq r_j, \ j = 2, ..., J \\ 0 & \text{for } r > r_J \end{cases} \cdot$$

Consider a market for credit where there is adverse selection such that all projects with returns strictly below R_j are rationed. Firms j through J obtain capital at an interest rate $r^* \in (r_{j-1}; r_j]$ where demand and supply intersect. Within that interval, any lower interest rate would yield a lower return and excess demand $S(i(r)) < I(r)$ while higher rates are associated with excess supply $S(i(r)) > I(r)$. Although a situation of excess supply implies higher returns $i(r)$ by $dS(i)/di > 0$, there is no means available to banks that actually generates more of this return, i.e. no residual demand. Suppose further that $i(r) < i(r^*)$ for all $r < r^*$ such that higher returns are in fact not possible through lower debt interest rates. This ensures that r^* is in fact a unique-price-equilibrium. The converse case of equilibria with multiple prices is delegated to one of the examples found in the following Subsection.

With probabilities p_j', we obtain $S(i(r_j)) = S(i(r_j'))$ as well as $I(r_j) = I(r_j')$ for any critical interest rate r_j. Therefore, we know specifically that there is still excess demand when firm $j-1$ is on the verge of exiting the market due to $S(i(r_{j-1})) < I(r_j) \Rightarrow S(i(r_{j-1}')) < I(r_j')$.[7] Similarly, before firm j exits the market, there is an intersection of demand and supply: $S(i(r_j)) > I(r_j) \Rightarrow S(i(r_j')) > I(r_j')$. Finally, i is fixed at that value which obtains $S(i) = \sum_{k=j}^{J} N_k B$. This concludes the proof for neutrality of success probabilities in the case of adverse selection.

The above neutrality result is in stark contrast to the findings of Stiglitz & Weiss

[7] $S(i(r_{j-1})) < I(r_{j-1})$ would suffice for no unique equilibrium to the left of that under consideration, but would still allow for a two-prices-equilibrium to happen.

(1992). This is due to the fact that, there, the assumption of a constant expected return is dropped, which makes sense for their interpretation of changes in p_j that refers to business cycles. We, on the other hand, consider a different change of capital markets away from or towards riskiness per se. Our results have in common that it is only relative probabilities that matter.

5.2 Adverse Selection and Two-Price-Equilibria: Examples

We stick with to example from Subsection 4.2. Using (9), we can calculate interest rates when capital demand of each type drops to zero. They take on values of

$$r_1 = \frac{100 - 0.2 \times 50}{0.8 \times 80} - 1 = \frac{13}{32}$$

and

$$r_2 = \frac{100 - 0.5 \times 50}{0.5 \times 80} - 1 = \frac{7}{8}.$$

They define when firms leave the market and, thereby, the structure of capital demand. In order to obtain a regular (unique-price) adverse selection equilibrium, we change firm numbers to $N_1 = 150$ and $N_2 = 350$. The former case is treated below as an illustration of the two-price-equilibrium (see also Arnold et al., 2014). As critical loan interest rates change in the same way when altering p_j's, i.e., turning to the safer variant of this market (with $p'_j = 1.1 p_j,\ j = 1, 2$), critical interest rates become

$$r'_1 = \frac{100 - 0.12 \times 50}{0.88 \times 80} - 1 = \frac{59}{176}$$

and

$$r'_2 = \frac{100 - 0.45 \times 50}{0.55 \times 80} - 1 = \frac{67}{88}$$

independently of the N_j's.

5.2.1 Adverse Selection: Example

With the above numbers (lengths of the continua) of firms, $N_1 = 150$ and $N_2 = 350$, capital demand is

$$I(r) = \begin{cases} (150+350) \times 80 = 40\,000, & \text{for } r \leq \frac{13}{32} \\ 350 \times 80 = 28\,000, & \text{for } \frac{13}{32} < r \leq \frac{7}{8} \\ 0, & \text{for } r > \frac{7}{8} \end{cases} .$$

Using lender profits from (11), we can calculate generated returns to the bank depending only on r. They are

$$i(r) = \begin{cases} 0.59r - \frac{123}{800}, & \text{for } r \leq \frac{13}{32} \\ 0.5r - 0.1875, & \text{for } \frac{13}{32} < r \leq \frac{7}{8} \end{cases} .$$

Inserting the critical rates, we have $i(r_1) = 11/128$ and $i(r_2) = 0.25$. The corresponding levels of capital supply are $S(i(r_1)) = 17,187.5$ and $S(i(r_2)) = 50,000$. The equilibration of demand and supply can clearly only happen in the range $(r_1; r_2]$ here as $S(i(r_1)) < I(r_1)$ but $S(i(r_2)) \geq I(r_2)$. Indeed, we obtain a (single-price) equilibrium with

$$28\,000 = 200\,000 \times (0.5r - 0.1875) \Leftrightarrow r^* = 0.655.$$

The equilibrium is therefore characterized by adverse selection: only the owners of riskier projects (type 2) obtain capital at a relatively high loan interest rate. A graphic illustration is provided in Figure 2.

If we increase success probabilities in a mean-preserving way by 10% here, we get returns of

$$i'(r) = \begin{cases} 0.649r - \frac{1\,053}{8\,000}, & \text{for } r \leq \frac{59}{176} \\ 0.55r - \frac{27}{160}, & \text{for } \frac{59}{176} < r \leq \frac{67}{88} \end{cases} .$$

Returns obtain identical maximum values of $i(r_1') = 11/128 = i(r_1)$ and $i(r_2') = 0.25 = i(r_2)$ as before. Hence, at r_1', capital supply falls short of capital demand by

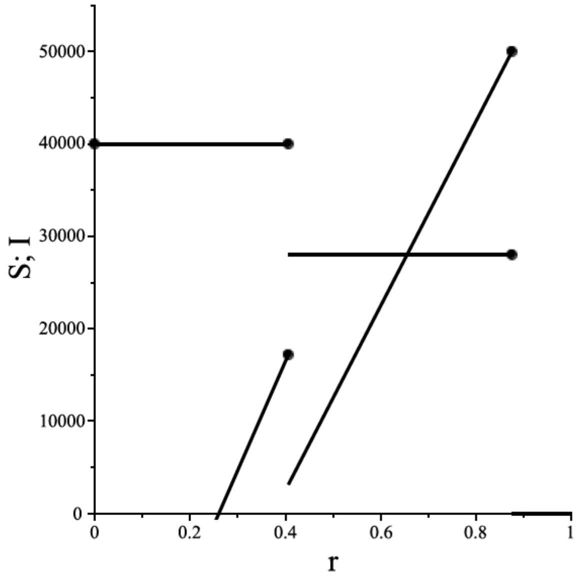

Figure 2: Equilibrium with adverse selection

the same amount as before at r_1. The equilibrium is located in the second segment of capital demand where the equilibrium of the latter with supply accounts to

$$28\,000 = 200\,000 \times (0.55r - \frac{27}{160}) \Leftrightarrow r^{*\prime} = \frac{247}{440}.$$

This equilibrium suffers from the same imperfect information frictions since the same types of borrowers as in the benchmark, namely those with higher success probabilities, are discouraged from applying for a loan due to too high repayment obligations – there is still adverse selection.

We illustrate this new equilibrium by the green curves in Figure 3 where we simply add the new lines for capital demand and supply to Figure 2. The fact that it looks like a compression of the (black) reference case is no mere coincidence: Graphically, when changing all p_j simultaneously by factor $(1 + x)$, we always stretch $(x < 0)$ or clinch $(x > 0)$ the entire diagram.

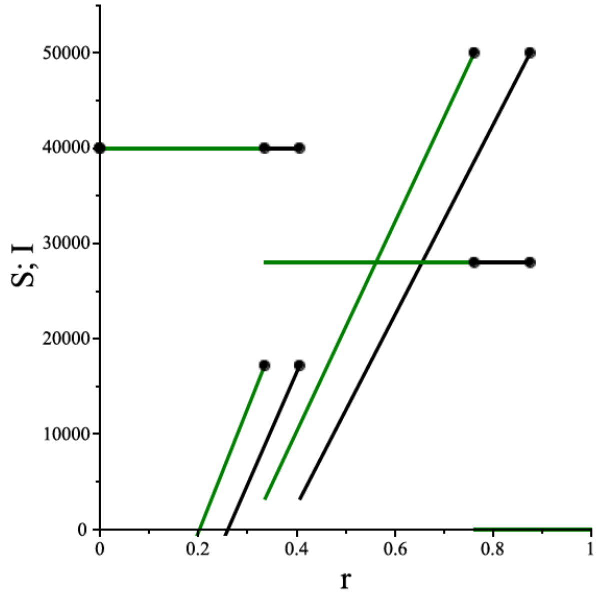

Figure 3: Equilibria with adverse selection under different probability regimes

5.2.2 Two-Price-Equilibrium: Example

Returning to $N_1 = N_2 = 250$, we obtain capital demand

$$I(r) = \begin{cases} (250+250)\times 80 = 40\,000, & \text{for } r \leq \frac{13}{32} \\ 250 \times 80 = 20\,000, & \text{for } \frac{13}{32} < r \leq 0.875 \\ 0, & \text{for } r > 0.875 \end{cases} .$$

As in the previous example, we use lender profits from (11) to calculate banks' returns as a function of r exclusively:

$$i(r) = \begin{cases} 0.65r - \frac{21}{160}, & \text{for } r \leq \frac{13}{32} \\ 0.5r - 0.1875, & \text{for } \frac{13}{32} < r \leq 0.875 \end{cases} .$$

Consequently, the critical rates give rise to maximum possible returns for given amounts of selection of $i(r_1) = 17/128$ and $i(r_2) = 0.25$, which can pool capital supply of $S(i(r_1)) = 26\,562.5$ and $S(i(r_2)) = 50\,000$, respectively.

The naïve approach to equate demand and supply in their second segments does not yield an equilibrium here. If banks lent capital only to firms of type 2 at the rate $r^* = 0.575$ (at which demand and supply intersect), profits could be raised by charging $r_1 = 13/32$ from project-1-owners that have not applied for a loan so far. Hence, lenders will charge r_1 right away. In this process, credit is rationed to an extent such that residual demand can be exactly satisfied by pooling savings at a higher interest rate such that the same rate of return is secured. A lower rate would lead banks to skip this second round, while a higher one is not achievable due to competitive pressure. Both firm types are rationed in proportion to their mass. The commensurate distribution of rationing happens with certainty as there is a continuum of firms of each type. Afterwards, some firms of the second type (those who turned out unlucky in the first round) would still demand capital even at a higher rate. Banks can satisfy this residual demand by equating it to residual supply as long as the low-interest round indeed happened first. Hence, by charging $\tilde{r}$ such that $i(\tilde{r}) = i(r_1)$, no residual demand is left and returns are not diminished. This is precisely the case when

$$\frac{17}{128} = 0.5r - 0.1875 \Leftrightarrow \tilde{r} = \frac{41}{64}.$$

In this equilibrium, all firms of type 2 obtain capital. Each firm of type 1 is only provided a loan with probability $\tilde{S}/(N_1+N_2)B$, where $\tilde{S}$ denotes allocated capital from the first round. We illustrate the equilibrium graphically in Figure 4.

After the increase in the p_j's, capital demand jumps at lower values of r and is otherwise unaltered:

$$I'(r) = \begin{cases} 40\,000, & \text{for } r \leq \frac{59}{176} \\ 20\,000, & \text{for } \frac{59}{176} < r \leq \frac{67}{88} \\ 0, & \text{for } r > \frac{67}{88} \end{cases}.$$

Dividing lender profits from (11) by B and subtracting 1 gives returns as

$$i'(r) = \begin{cases} 0.715r - \frac{171}{1600}, & \text{for } r \leq \frac{59}{176} \\ 0.55r - \frac{27}{160}, & \text{for } \frac{59}{176} < r \leq \frac{67}{88} \end{cases}.$$

Obtainable returns and corresponding levels of capital remain the same as before the change in success probabilities: $i'(r_1') = 17/128 = i(r_1)$, $i'(r_2') = 0.25 = i(r_2)$,

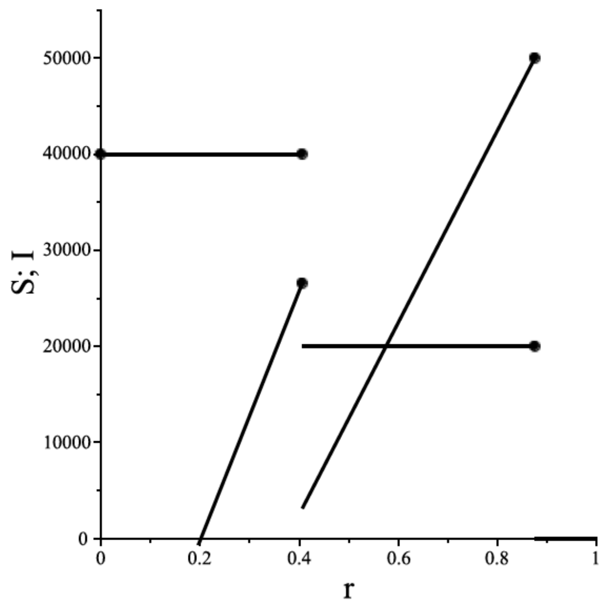

Figure 4: Equilibrium with two prices

$S(i'(r_1')) = 26\,562.5 = S(i(r_1))$ and $S(i'(r_2')) = 50\,000 = S(i(r_2))$. As capital supply again enters the gap where capital demand jumps at $r_1' = 59/176$, we still obtain an equilibrium with two rounds of capital allotment. At r_1', equal portions of firm types receive capital. The remaining type-2 firms apply again at

$$\frac{17}{128} = 0.55r - \frac{27}{160} \Leftrightarrow \tilde{r}' = \frac{193}{352}$$

for which they all receive capital. It is still half of the type-1 borrowers that do not get their desired capital. Hence, the equilibria under both probability regimes are equivalent in real terms.

To see that credit rationing is in fact of identical magnitude in both cases, one can also consider Figure 5. There, the vertical distance between capital demand and supply at r_1 and r_1', respectively, is the same.

5.3 Generalization

The assumption followed so far that probability changes are always equiproportionate with factor $1 + x$ may turn out immaterial in some applications. We devote this slide-in subsection to a generalization on how the neutrality result

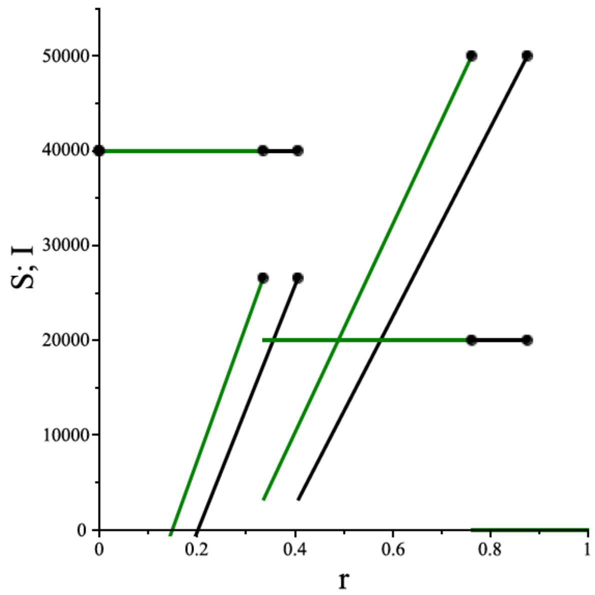

Figure 5: Equilibria with two prices under different probability regimes

can hold even if only one success probability changes. That is, there exist cases in which even disproportionate probability changes leave both the kind of equilibrium (including the amount of rationing) and the savings interest rate unaltered. For ease of exposition, we restrict ourselves to $J = 2$ risk classes.

Suppose the capital market equilibrium is of the same qualitative structure as seen in Figure 2: There is adverse selection such that $j = 1$-firms do not apply for a loan. Any change in p_2 modifies the shape of capital supply in both segments of the graph. The reason for this is that the return function chnages in response to the average success probability being altered independently of whether both firms apply or only type 2. However, capital supply in the second segment remains more than sufficient to cover demand as the return still grows up to its global maximum of $E(R)/B - 1$. As the return function experiences a downward jump at r_1, it is sufficient to assume $S(i(r_1)) < I(r_1)$ in order to uphold the unique intersection of demand and supply in the second segment (preempting a two-prices-equilibrium at the same time). Both the savings interest rate and capital allotment are then unvaried. So we simply have to prove that there is a plausible interval of p_2''s for which $S(i(r_1)) < I(r_1)$ continues to hold.

The average success probability of a financed project in the two-type-case before adverse selection comes into play is

$$E[p|r \leq r_1] = \frac{N_1 p_1 + N_2 p_2}{N_1 + N_2}. \tag{15}$$

That probability plays a part for expected bank profits at r_1:

$$E(\pi^L | r = r_1) = E[p|r \leq r_1][(1 + r_1)B - C] + C. \tag{16}$$

Those are necessary to determine the return generated by lenders which they pass on as the savings interest rate. More precisely, $S(i(r_1)) < I(r_1)$ can be expressed as

$$S\left(\frac{E(\pi^L | r = r_1)}{B} - 1\right) < (N_1 + N_2)B.$$

Inserting (15) as well as (16) and rearranging terms yields

$$p_2 < \frac{(N_1 + N_2)\{[1 + S^{-1}((N_1 + N_2)B)]B - C\} + C}{N_2(1 + r_1)B} - \frac{N_1}{N_2} p_1. \tag{17}$$

Inequality (17) reveals that there are most definitely values in the neighbourhood of the initial p_2 which obtain the neutrality result. Firstly, we can reduce p_2 and make it arbitrarily close to zero as only a too high p_2' would be problematic.[8] Secondly, as the initial p_2 must have fulfilled (17), there must also exist some larger values such that it still holds ($p_2' = p_2 + \varepsilon$ where ε is sufficiently close to zero). To sum up, we can conclude that, denoting the RHS of (17) as p_2^*, any $p_2' \in (0, p_2^*) \neq \emptyset$ for given p_1 obtains a neutrality result in the sense discussed above. Hence, it becomes evident that our result also extends to disproportionate probability changes.

By a similar logic, one can also obtain that, for constant p_2, we can make p_1 arbitrarily close to unity or reduce it to a value just above some p_1^* without altering the adverse-selection-structure of the benchmark equilibrium (including the interest rate on savings and capital allotment). This serves to strengthen the generality of our neutrality result.

[8]We must, however, exclude $p_2 = 0$ because this would necessitate $R_2 \to +\infty$ for constant $E(R) > 0$.

6 Hidden Actions

Equilibria characterized by moral hazard require a different kind of information imperfection than the one employed so far: the market side at an informational advantage must be able to make some hidden choice. To incorporate this, the model will be modified in a straightforward way.

6.1 Equilibrium with Moral Hazard

To depict hidden action, we no longer consider firms to be endowed with one single project. Rather, each of the N firms is capable of conducting all J projects. While it would be in the lenders' interest to secure the implementation of project 1, they cannot force borrowers to do so directly. The project choice depends on the charged loan interest rate r. With firm profits conditional on the choice of project j given by (1), we can immediately conclude that each firm would always choose the riskiest project as $E(\pi_j^F) \geq E(\pi_{j+1}^F),\ j = 1, ..., J-1$ necessitates

$$r \leq \frac{C}{B} - 1 < 0.$$

In words, incentivizing the conduction of safe projects requires banks to actually pay corporates for doing so.[9] As then $i < 0$, no capital supply can be attracted. The only possible equilibrium entails project choice J by all firms.

To obtain equilibria entailing non-trivial moral hazard (like the one described above), we alter project returns by $\Delta R_1 > \Delta R_2 > ... > \Delta R_J$ the sign of which is irrelevant. On the outset, the only restriction we have to impose on the ΔR_j's is that they are not too high such that the resulting R_j still obey $R_1 < R_2 < ... < R_J$.[10] We denote expected returns by $E(R_j)$ and define $\alpha_j > 1$ as a mark-up

[9] It is worth noting that firms will in fact have no incentive to switch back to the riskiest project. This is due to the fact that a riskier project choice would amplify the loss in case of a failure as the cheap repayment is then more likely to be missed out on.

[10] We abstain from introducing a new notation for payoffs here in order to warrant readability.

factor which states by how much project j is, on average, better than $j+1$:

$$E(R_j) = \alpha_j E(R_{j+1}),\ \ j = 1, ..., J-1. \tag{18}$$

By (1) and (18), we know that a safer project is preferred to the next-riskier one by borrowers as long as

$$r \leq \frac{(\alpha_j - 1)E(R_{j+1}) + (p_j - p_{j+1})C}{(p_j - p_{j+1})B} - 1 \equiv r_j\ (> 0),\ \ j = 1, ..., J-1. \tag{19}$$

Firms stop demanding capital if not even the riskiest project is worth financing anymore due to a too high interest burden. $E(\pi_J^F) \geq 0$ requires

$$r \leq \frac{E(R_J) - (1 - p_J)C}{p_J B} - 1 \equiv r_J. \tag{20}$$

To ensure that both moral hazard per se as well as the switch-inducing rates in (19) and (20) are meaningful for our model, we have to assume that those rates are increasingly ordered. That is, we need $r_1 < r_2 < ... < r_J$ or, more formally, $r_j < r_{j+1}, j = 1, ..., J-1$. This condition yields a threshold for each α_j but the last one:

$$\alpha_j < \frac{p_j - p_{j+1}}{p_{j+1} - p_{j+2}}(1 - \frac{1}{\alpha_{j+1}}) + 1, j = 1, ..., J-2. \tag{21}$$

As the above threshold is always greater than one (as long as $\alpha_{j+1} > 1$, which has to be true), it can never be logically inconsistent. Further, it depends positively on α_{j+1}, which tells us that, in comparative terms, avoiding risk has to become sufficiently less attractive the fewer risk is already taken in order to be worth entertaining more of it as the burden of repayment grows. In other words, each additional risk taken has to be disproportionately more promising in expectations than the previous risk increase. Whenever $r_{J-1} < r_J$ additionally, r_J constitutes the unique global maximum of the return function again. This is the case whenever the gain in surplus from avoiding the worst type of risk is not too high:

$$\alpha_{J-1} < \frac{p_{J-1}}{p_J} - \frac{p_{J-1} - p_J}{p_J}\frac{C}{E(R_J)}. \tag{22}$$

If we were to restrict oureselves to $J = 2$ risk classes, (22) alone would suffice. Neglecting collateral would obviate the above derivations of (21) and (22) because then, no matter what, more risk eventually becomes attractive as r rises: there is never anything to loose from conducting a project but always some chance of positive profits whenever $R_j > (1+r)B$, which always holds for $j+k, k \geq 1$ if it holds for given j, i.e. if even more risk is being taken, but not necessarily the other way round (see also Arnold, 2020, pp. 246-248).

We can now determine firms' actions depending on r. They choose project 1 for $r < r_1$, switch to project 2 there, which they conduct up until $r_2 (> r_1)$ where they switch to project 3, and so on until they quit demanding capital at r_J. Note that every firm acts identical as they all choose from the same pool of projects. Formally, capital demand satisfies

$$I(r) = \begin{cases} NB, & \text{for } r \leq r_J \\ 0, & \text{for } r > r_J \end{cases}.$$

Bank profits are given by (2) where p_j is determined uniquely as the one for which $r_{j-1} < r \leq r_j$ holds. Thus, returns are

$$i(r) = \begin{cases} p_1(1+r) + (1-p_1)\frac{C}{B} - 1, & \text{for } r \leq r_1 \\ p_j(1+r) + (1-p_j)\frac{C}{B} - 1, & \text{for } r_{j-1} < r \leq r_j \ j = 2, ...J \end{cases}. \tag{23}$$

Inserting critical interest rates (those where switches happen) into (23), we obtain

$$i(r_j) = \frac{C}{B} + \frac{p_j}{p_j - p_{j+1}} \frac{(\alpha_j - 1)E(R_{j+1})}{B} - 1, \ j = 1, ..., J-1$$

and, of course, the entire project return at r_J:

$$i(r_J) = \frac{E(R_J)}{B} - 1.$$

Now, we turn to the influence of p_j again. Altering them all simultaneously by $(1+x)$ and R_j by $1/(1+x)$ just as in Section 5, we obtain $E(R'_j) = E(R_j), \ j = 1, ..., J$. The switch-inducing loan rates from (19) become

$$r'_j = \frac{(1+x)(p_j - p_{j+1})C + (\alpha_j - 1)E(R_{j+1})}{(1+x)(p_j - p_{j+1})B} - 1, \ j = 1, ..., J-1.$$

Similarly, we get

$$r'_J = \frac{E(R_J) - (1+x)(1-p_J)C}{(1+x)p_J B} - 1$$

from (20). Inserting both into (23) immediately yields

$$i'(r'_j) = i(r_j),\ \ j = 1, ..., J.$$

Hence, whenever the loan interest rate is reached at which corporates alter their choice of project conduction, the return thus generated is already fixed. Due to constant capital demand in the range of acceptable loan rates it is therefore certain that, if credit rationing arises, its extent is numerically identical under both distributions of risk. By the same logic as in the previous sections, the equilibrating i determines the conducted project uniquely.

6.2 Moral Hazard Equilibrium: Example

In order to make our above example fit into the variant of the model where moral hazard plays a role, we need $E(R_1) > E(R_2)$. Therefore, we now use $R_2 = 170$ such that $E(R_1) = 100 > 85 = E(R_2)$. By (18), this gives $\alpha_1 = 20/17$. This, in turn, allows us to calculate the switching rate beyond which project 2 is financed via (19) as

$$r_1 = \frac{(0.8 - 0.5) \times 50 + (\frac{20}{17} - 1) \times 85}{(0.8 - 0.5) \times 80} - 1 = 0.25.$$

Firms stop demanding capital if credit interest rates go beyond

$$r_2 = \frac{85 - 0.5 \times 50}{0.5 \times 80} - 1 = 0.5$$

by (20). Until then, capital demand is constant at $NB = 40\,000$. The return from financing follows (23), i.e.

$$i(r) = \begin{cases} 0.8(1+r) + 0.2\frac{50}{80} - 1, & \text{for } r \leq 0.25 \\ 0.5(1+r) + 0.5\frac{50}{80} - 1, & \text{for } 0.25 < r \leq 0.5 \end{cases}$$

which gives rise to $i(r_1) = 0.125$ and $i(r_2) = 0.0625$. Consequently, capital supply is equal to $S(i(r_1)) = 25\,000$ and $S(i(r_2)) = 12\,500$. From this we know that there

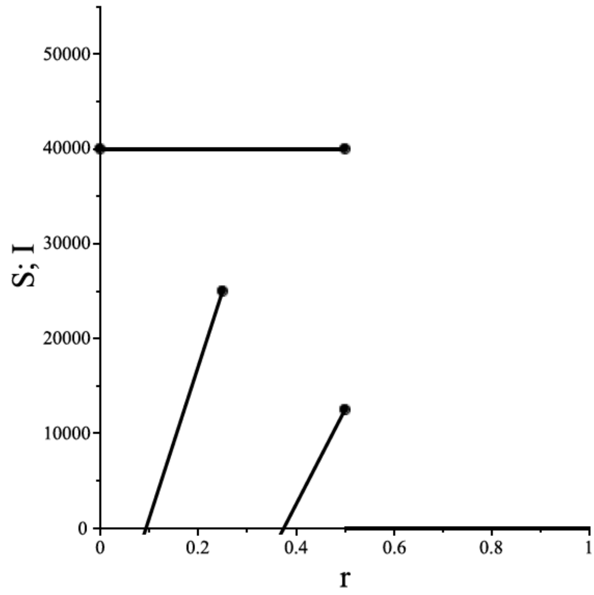

Figure 6: Equilibrium with moral hazard

can be no equilibrium without credit rationing. Rather, in equilibrium, banks generate the maximum possible return of 12.5%, project 1 is conducted and credit is rationed. A continuum of length $15\,000/80 = 187.5$ of firms is denied a loan. We depict this equilibrium graphically in Figure 6 using the usual formatting. Interestingly, full allotment would be part of an equilibrium if project 2 did not exist or could be ruled out contractually: Firms would then tolerate loan rates up to 0.40625 such that supply and demand can be equilibrated at $r = 11/32$.

Now, let p_1 and p_2 rise by 10% again (with $R_1' = 2\,000/11$ as before and $R_2' = 1\,700/11$). The new critical interes rates are

$$r_1' = \frac{(0.88 - 0.55) \times 50 + (\frac{20}{17} - 1) \times 85}{(0.88 - 0.55) \times 80} - 1 = \frac{17}{88}$$

and

$$r_2' = \frac{85 - 0.45 \times 50}{0.55 \times 80} - 1 = \frac{37}{88}.$$

Consequently, returns are

$$i(r) = \begin{cases} 0.88(1+r) + 0.12\frac{50}{80} - 1, & \text{for } r \le \frac{17}{88} \\ 0.55(1+r) + 0.45\frac{50}{80} - 1, & \text{for } \frac{17}{88} < r \le \frac{37}{88} \end{cases}$$

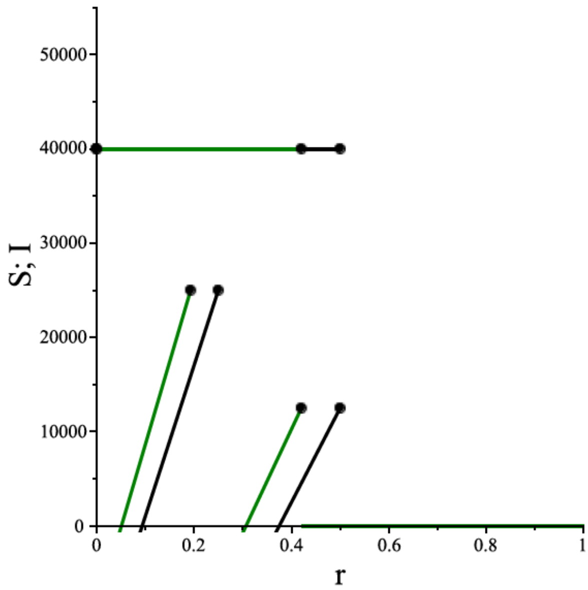

Figure 7: Equilibria with moral hazard under different probability regimes

and achieve local maxima at $i'(r_1') = 0.125 = i(r_1)$ and $i'(r_2') = 0.0625 = i(r_2)$. Of course, capital supply at those rates equals its previous levels. We therefore obtain an equilibrium with $r^{*\prime} = 17/88$, $i^{*\prime} = 0.125$ and credit rationing of 15 000 units of capital (187.5 firms) where all firms decide on project 1. The new equilibrium is depicted in Figure 7 in green as a compression of the former one (in black).

7 Financing projects via shares

Financing via shares may bring about an easy solution to the informational inefficiencies arising in the SW model. The result that underinvestment can be mitigated this way dates back to DeMeza & Webb (1987). For either variant of information imperfection, we can propose financing via shares as an alternative source of capital. The results will, however, turn out to be simple to the point of triviality. For a given, common level of collateral, adverse selection and moral hazard can be brought about in the following way. If firms pledge some fraction s of their net worth in exchange for the provision of capital B, their changes in profits starting from no involvement of the lender[11] follow

$$E(\pi_j^F) = (1-s)((E(R_j)+C) - C.$$

Applying this to the hidden information case reveals that the critical proportion any firm is willing to give up has to satisfy $E(\pi_j^F) \geq 0$, hence

$$s \leq \frac{E(R_j)}{E(R_j)+C} \equiv s_j, \ j = 1, ..., J.$$

With equal $E(R)$ as in Section 5, firms do not differ at all. With higher $E(R_j)$ for safer j, the riskier firms are actually those that are rationed first. This essentially posits a case of advantageous selection, a term coined by de Meza & Webb (1999) (although, there, the context is overinvestment). It can be turned back into adverse selection by assuming $E(R_1) < E(R_2) < ... < E(R_J)$ instead.
For the case of hidden actions, firms will always choose the project yielding the highest expected return. Depending on the ranking of the $E(R_j)$'s, this will result in moral hazard or moral harmony (or simply total indifference):

$$E(\pi_j^F) \gtrless E(\pi_{j+1}^F) \Leftrightarrow E(R_j) \gtrless E(R_{j+1}), \ j = 1, ..., J-1.$$

Either viewpoint can be used to yield an equilibrium with moral hazard or adverse selction. However, it is clear that the p_j's do not have any influence on it whatsoever because they only affect tolerated share issues via $E(R_j)$ which is assumed to be constant.
Differences in firm 'quality' that can yield a rationing-type equilbrium are obtained once the collateral is allowed to vary across corporates. Therefore, one would have to model risky firms as those who are able to pledge lower collateral.[12] If one then even allows C_j to vary when switching from the p_j- to the p'_j-regime (assuming a positive association of both variables), the neutrality result could fail

[11] It is perhaps more appropriate to speak of a shareholder rather than a bank here.

[12] If risky firms were the ones able to pledge a higher value of C, the term "risky" would simply become inadequate. Additionally, they suffer more from the danger of loosing it due to their low p_j.

to hold and additional safety would even exacerbate the informational problems of the market.

8 Conclusion

Using the Stiglitz-Weiss model (1981), we investigate the role of project riskiness for market equilibria. Expected project returns are left constant. In this way, we isolate the effect of success probabilities per se from others resulting from growth or business cycles. The ongoing transition to more green markets may be considered a current example.

Looking at complete markets as a benchmark, we obtain a neutrality result: the amount of rationing and the savings interest rate obtained in a capital market equilibrium is unchanged. This motivates a further look into cases of asymmetric information.

For a market of hidden information, we find that problems of unidentifiability of firms by banks cannot be mitigated by a safer pool of borrowers. Rather, credit interest rates charged per risk class simply fall (rise for a riskier pool) in such a way that savings interest rates remain at their pre-risk change level. This leaves the volume of investments financed in equilibrium constant.

A similar result holds if firms are characterized by their ability to take hidden actions: again, the only change happening after an alteration of success probabilities is in the interest rate on loans. Firms finance the same projects as before and credit rationing remains at a numerically identical level.

Considering different ways of financing such as a stock market can also maintain the neutrality result. However, there are variants of the model where this is not the case, for example when project riskiness influences pledged collateral. We consider the influence of both collateral choice and financing method to be interesting avenues of further research on this topic.

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