4 Adverse Selection (Screening) in the PA model

- Before (Anke): concerned with whether some given outcome (efficient, balanced) is implementable
  now: look for the constrained-optimal outcome from a certain individual's point of view. We call this agent the 'principal', and seek to find the outcome that maximizes his utility. Problem: Principal doesn't know the agent's relevant characteristics, his type.

- Huge number of applications. Selection: monopolistic price discrimination (monopolistic producer doesn't know customer preferences); optimal taxation (when deciding on income tax schedule, government doesn't know individual opportunity costs of labor market participation); procurement (buyer doesn't know seller's production costs); financing and lending (bank doesn't know client risk type) etc.

- Important: We need to assume that the *uninformed* party (the principal) makes the contract offer. Results do not extend to more general bargaining procedures.

The Two-Type Case

- consider example: monopolistic price competition

- two risk-neutral parties, a monopolistic seller (the principal P) and a single buyer (the agent A)

- Utility (profit) of seller is

$$U_P(t,x) = t - cx, \qquad c \geq 0, \tag{4.1}$$

  where $x \in [0,\infty]$ = quantity of good produced by P, and

– utility of the consumer is

$$U_A(x;\theta) = \theta v(x) - t, \quad v' > 0, v'' < 0 \tag{4.2}$$

where $\theta \in \{\theta_l, \theta_H\}, \theta_l < \theta_h$ and $p = Prob\{\theta = \theta_H\} > 0$

– Note: agent's preferences satisfy the *single crossing property*:

$$u(x,\theta_h) - u(x,\theta_l) = (\theta_h - \theta_l)v(x) \quad \text{strictly increases in } x \tag{SCP}$$

4.1 Perfect Information (First Best)

– if θ is commonly observable → contracts can be made contingent on the type of consumer, (t_i, x_i), $i = l, h$

– P solves $\max_{\{x_i,t_i\}} \quad t_i - cx_i$

$$\text{subject to} \quad U_A = \theta_i v(x_i) - t_i \geq 0, \; i = l, h \tag{PC_i}$$

– solution is characterized by (see Figure 1)

$$\theta_i v'(x_i^{FB}) = c \quad i = l, h$$

and $u_A(x_i^{FB}, t_i^{FB}) = 0$ from (PC_i), so

$$t_i^{FB} = \theta_i v(x_i^{FB})$$

→ if the monopolist can perfectly price discriminate, she will offer the quantities $x_h^{FB} > x_l^{FB}$ and extract the entire consumer surplus

→ Note: this outcome is *efficient* (Pareto optimal)

4.2 Imperfect Information

– θ is A's private information. As a consequence: $\{(y_i^{FB}, t_i^{FB})\}_{i=l,h}$ no longer implementable or *incentive compatible* (see Figure 2). Formally:

$$u_A(x_h^{FB}, t_h^{FB}; \theta_h) = \theta_h v(x_h^{FB}) - t_h^{FB} = 0$$
$$< (\theta_h - \theta_l) v(x_l^{FB}) = \theta_h v(x_l^{FB}) - t_l^{FB} = u_A(x_l^{FB}, t_l^{FB}; \theta_h)$$

$\rightarrow$ high-preference type would choose contract for low-demand agent

– Note: P can still achieve efficient purchases by offering the menu (x_l^{FB}, t_l^{FB}) and $(x_h^{FB}, t_h^{FB} - \psi^{FB})$ where

$$\psi^{FB} = \psi(x_l^{FB}) = (\theta_h - \theta_l) v(x_l^{FB})$$

$\rightarrow$ But: only if the high-type agent pays far less than t_h^{FB} to the principal! Means: the θ_h-type agent earns a *rent* (see Figure). P doesn't like this.

4.3 The Second Best Optimum

– Q: What is the optimal (second-best) allocation and the corresponding mechanism?

– Revelation Principle (see Anke) $\rightarrow$ restrict ourselves w.l.o.g. to direct mechanism/ menu of contracts $\{(x(\hat{\theta}), t(\hat{\theta}))\}$ which is incentive compatible, i.e. $\hat{\theta} = \theta$

– the principal chooses $\{(x_i, t_i)_{i=l,h}\}$ so as to

$$\max_{(x_i, t_i)} \quad E(U_P) = (1-p)\big[t_l - cx_l\big] + p\big[t_h - cx_h\big]$$

$$\text{subject to} \quad U_A(\cdot, \theta_i) = \theta_i v(x_i) - t_i \geq 0, \qquad i = l, h \qquad (PC_i)$$

$$\text{and} \qquad \theta_l v(x_l) - t_l \geq \theta_l v(x_h) - t_h \qquad (IC_l)$$

$$\theta_h v(x_h) - t_h \geq \theta_h v(y_l) - t_l. \qquad (IC_h)$$

<u>Claim:</u> the optimal contract has the following properties

1. (PC_l) is binding $\rightarrow t_l = \theta_l v(x_l)$
2. (IC_h) is binding $\rightarrow t_h = t_l + \theta_h[v(x_l) - v(x_l)]$
3. $x_h \geq y_l$
4. (PC_h) and (IC_l) are slack (and, hence, can be ignored)

– using properties 1–4, P's max program reduces to

$$\max_{(x_h, x_l)} \quad (1-p)\big[\theta_l v(x_l) - cx_l\big] + p\big[\theta_h v(x_h) - cx_h - (\theta_h - \theta_l)v(x_l)\big]$$

– the FOC are

$$\theta_h v'(x_h^{SB}) = c \qquad \Rightarrow \qquad x_h^{SB} = x_h^{FB}$$

$$\theta_l v'(x_l^{SB}) = \frac{c}{1 - \frac{p}{1-p}\frac{\theta_h - \theta_l}{\theta_l}} > c \qquad \Rightarrow \qquad x_l^{SB} < x_l^{FB}.$$

– we get the optimal transfers from (PC_l) and (IC_h),

$$t_l^{SB} = \theta_l v(x_l^{SB}) \qquad \Rightarrow \qquad u_A(x_l^{SB}, t_l^{SB}; \theta_l) = 0$$

$$t_h^{SB} = \theta_h v(x_h^{SB}) - (\theta_h - \theta_l)v(x_l^{SB}) \quad \Rightarrow$$

$$u_A(x_h^{SB}, t_h^{SB}; \theta_h) = (\theta_h - \theta_l)v(x_l^{SB}) = \psi(x_l^{SB}) > 0.$$

4.4 Interpretation

- in order to ensure self-selection, P has to give a *rent* of $\psi(x_l) = (\theta_h - \theta_l)x_l$ to the θ_h-type agent, with $\psi' > 0$

- it is the necessity to pay this rent that makes it optimal for P to distort the quantity bought by the θ_l type agent downwards (see Figure)
- size of distortion depends on relative probability of a low-demand agent: $x_l^{SB} \longrightarrow x_l^{FB}$ as $p \longrightarrow 0$ and $x_l^{SB} = 0$ for sufficiently high p
- Output of h-type agent is same than in FB ('no distortion at the top'-property)
- P has perfect information *ex post* $\rightarrow$ further gains from trade to be realized in state θ_l since $x_l^{SB} < x_l^{FB}$
  $\rightarrow$ self selection requires commitment (no renegotiation)
- there is a non-linear tariff $t(x)$ that is equivalent to the second-best optimal contracts:

$$t = t_i^{SB} \text{ if } x = x_i^{SB} \qquad \text{and} \quad t = \quad \infty \quad \text{otherwise.}$$